# The effect of eye position on the projected stimulus distance in a binocular head-mounted display

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## ABSTRACT

During vergence eye movements, the effective separation between the two eyes varies because the nodal point of each eye is offset from the center of rotation. As a result, the projected distance of a binocularly presented virtual object changes as the observer converges and diverges. A model of eye and stimulus position illustrates that if an observer converges toward a binocular virtual stimulus that is fixed on the display, the projected stimulus will shift outward away from the observer. Conversely, if the observer diverges toward a binocular virtual stimulus that is fixed on the display, the projected stimulus will shift inward. For example, if an observer diverges from 25 cm to 300 cm, a binocular virtual stimulus projected at 300 cm will shift inward to 241 cm. Accurate depiction of a fixed stimulus distance in a binocular display requires that the stimulus position on the display surface should be adjusted in real-time to compensate for the observer's eye movements.

**Keywords**: vergence, distance, binocular, ocular, projected, head-mounted display

## 1. INTRODUCTION

In virtual environments, visual stimuli are often presented binocularly. Binocular displays have the advantage of allowing stereopsis, defined as the perception of depth arising from the separation of the two eyes[1]. As the eyes rotate horizontally towards one another (convergence), the separation between the key optical components, such as the pupils, decreases[2,3,4,5,6]. Conversely, the separation between the key optical components increases if the eyes horizontally rotate away from each other (divergence). If the distance to the image plane is optically designed so that the user has to accommodate (focus) far away (such as greater than 1 m), the variation in ocular separation is generally ignored because it has a relatively minor effect on the stability of a virtual object[7]. However, if the image plane distance is close (such as 25 cm), the variation in ocular separation can produce a substantial effect on the projected distance in space to the stereoscopic virtual object.

Although the effects caused by the variable ocular separation are critical for presenting stable virtual objects, the effects have not been fully examined in prior research on binocular HMDs. For example, a report that discussed the importance of eye tracking did not examine the errors arising if the eyes are assumed to be fixed[3]. A model of stereoscopic optics in a HMD was incomplete because the ocular separation for a given observer was assumed to be constant[8]. Similarly, a psychophysical study on the display of binocular virtual objects assumed that eye position was constant[9].

The current paper provides a more comprehensive examination of the effects of ocular separation on the projected position to a binocular virtual object presented in a HMD. The model of stereoscopic display geometry presented here illustrates that substantial shifts in the projected target position can occur if the target is located in a fixed position on the display surfaces.

## 2. QUANTITATIVE ANALYSIS

A quantitative analysis of eye position and stimulus position must be based on a specific ocular reference point. Ocular geometry defined in general terms, such as with respect to the eyes themselves[10], does not indicate the change in eyepoint that occurs as the observer changes vergence. One possible reference point for measuring eye and stimulus angles is the entrance pupil[11,12], defined as the image of the iris formed by the cornea[13]. However, the ray from the visual target to the entrance pupil cannot be extended along a line to a point on the retina because the ray is diverted at the crystalline lens toward the exit

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pupil[14]. Consequently, a ray from the target to the entrance pupil that is linearly extended to the retina does not coincide with the target position on the retina.

A more appropriate reference is the nodal point, defined as the point in the eye where unrefracted rays cross[15]. As shown in Figure 1, each eye has two nodal points (anterior and posterior). They are separated by only 0.1 mm according to Ogle[16] or 0.3 mm according to Bennett and Francis[17] and Hopkins[18].

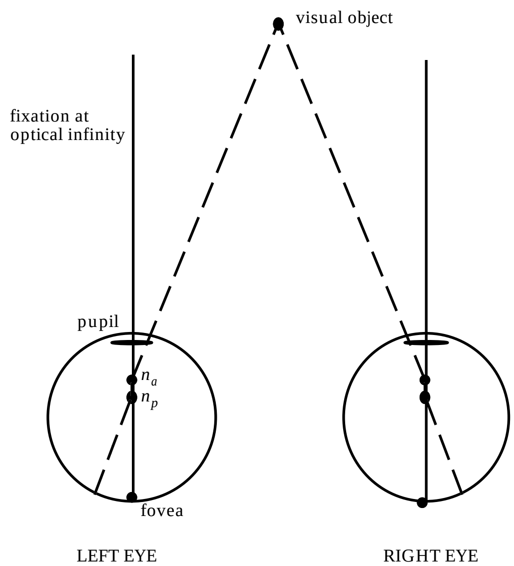


*Figure 1. Top–down view illustrating that a ray entering the anterior nodal point ($n_a$) is parallel to the ray leaving the posterior nodal point ($n_p$).*

Since the nodal points of each eye are so close together, one nodal point can be used to model the reduced visual optics of the eye[15, 19, 20, 21]. Using the nodal point as a reference, the vergence angle can be defined as the angle between the two visual axes, where the visual axis is defined as the line through the fovea and the nodal point[22]. Because entering and exiting rays through the nodal points are parallel[23], and because a single nodal point can be used to represent the optics of the eye, the visual axis can also be defined as the ray from the fovea to the fixation point[24, 21, 25], as shown in Figure 2. This axis can be assumed to pass through the ocular center of rotation[26].

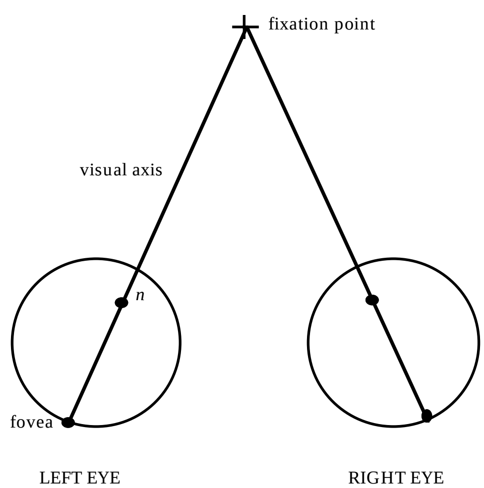


*Figure 2. Top–down view illustrating the visual axis that passes through the fixation point, nodal point, and fovea. Each eye can be modeled with a single nodal point (n).*

A top-view representation of the key ocular angles is shown in Figure 3. In this figure, the eyes are initially converged at angle $\gamma_1$ such that the observer is fixating point $f_1$. The stimulus is presented on the image plane such that it projects binocularly to position $s_1$ in space. To fixate the stimulus and obtain a single view of it, the observer must diverge. As the observer diverges to fixation point $f_2$, the nodal point moves from $n_1$ to $n_2$. As a result, the projected binocular position of the virtual object moves inward from $s_1$ to $s_2$ as the observer diverges from $f_1$ to $f_2$. The final fixation distance $f_2$ is identical to the final binocular stimulus distance $s_2$. At this point, the stimulus is binocularly fixated.

The angle subtended by the stimulus at the nodal points is called the binocular parallax (e.g., $\beta_1$). The other key angle associated with binocular vision is disparity, which can be categorized as absolute disparity and relative disparity[11]. Absolute disparity is the difference between the vergence angle of the eye and the angle subtended by a visual target at the nodal points. For example, when the observer is fixating point $f_1$, the absolute disparity is $\beta_1 - \gamma_1$. When the observer is fixating point $f_2$, the absolute disparity is zero because the fixation point is identical to the stimulus position. Relative disparity is the difference between the angles subtended by two visual targets at the nodal points[27, 28].

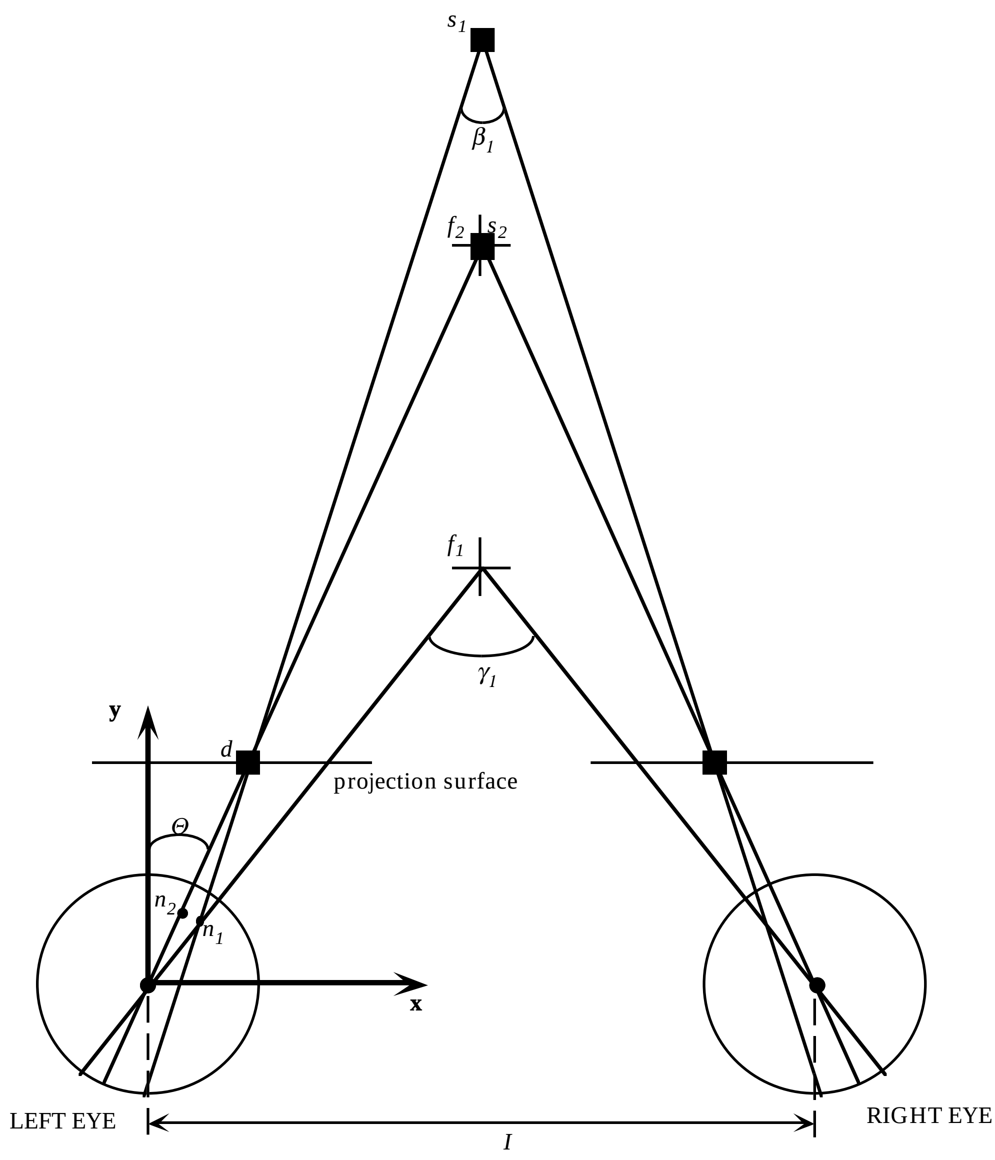


*Figure 3. Top-view illustrating a virtual target positioned at point d on a display surface. The initial fixation point is $f_1$, the initial nodal point position is $n_1$, and the initial projected position of the stimulus is $s_1$. To fuse the stimulus, the observer diverges to point $f_2$. As the observer diverges, the projected position of the stimulus shifts to point $s_2$. The final nodal point position is $n_2$. The distance between the centers of rotation is I. The x–y reference axis is fixed to the center of rotation of the left eye. This figure is not drawn to scale.*

Several independent equations are necessary to quantify the effects of vergence on the projected location of a virtual target. The linear variables in these equations are defined in terms of their distances to the x-y reference axis, which is defined as fixed to the center of rotation of the left eye. As shown in Figure 3, movements of the eye can be treated kinematically as though the center of rotation is fixed in the socket[21, 26]. The initial vergence position $\gamma_1$ can be found from the initial fixation distance $f_{1y}$ and the interocular separation ($I$) between the centers of rotation of the eyes.

$$\tan(\gamma_1 / 2) = \frac{(I / 2)}{f_{1_y}}) \tag{1}$$

Given the radius ($r$) from the center of rotation to the nodal point, the corresponding nodal positions ($n_{1x}$ and $n_{1y}$) also can be found.

$$n_{1_x} = r\sin(\gamma_1 / 2) \tag{2}$$
$$n_{1_y} = r\cos(\gamma_1 / 2) \tag{3}$$

Given the initial projected distance to the stimulus ($s_{1y}$) and the distance to the image plane ($d_y$), the stimulus horizontal position on the image plane ($d_x$) can found using similar triangles.

$$\frac{d_x - n_{1_x}}{d_y - n_{1_y}} = \frac{(I / 2) - n_{1_x}}{s_{1_y} - n_{1_y}} \tag{4}$$

The final fixation distance ($f_{2y}$), which is the final stimulus distance, also can be found from similar triangles.

$$\frac{d_x}{d_y} = \frac{(I / 2)}{f_{2_y}} \tag{5}$$

A natural issue arising from stereoscopic displays is the effect of vergence on the projected distance to the binocular visual stimulus. A quantitative analysis illustrating these effects is shown in Figure 4. In this figure, the vertical axis shows the projected distance to a binocularly presented virtual object as the observer changes vergence, and the horizontal axis shows the distance at which the observer is converging. In the real world, the lines in this figure remain horizontal. In other words, the distance to a real object is fixed regardless of the observer's vergence position. In a virtual environment, the projected distance to a virtual object varies with vergence position. The values on this graph are based on three fixed parameters: the interocular separation ($I$) between the centers of rotation of the eyes, the radius ($r$) from the center of rotation to the nodal point, and the image plane distance ($d_y$) to the center of rotation. For the separation parameter $I$, a value of 6.2 cm was selected because it is the mean of the typical interpupillary distance (IPD) range (5.0 cm to 7.4 cm)[29]. Because the IPD is measured when the eyes are parallel[30], the separation parameter $I$ is assumed equal to the IPD. For the nodal distance parameter r, a value of 0.6 cm was selected because the center of rotation is 1.3 cm from the cornea[24, 31] and the nodal point is 0.7 cm from the cornea[15] (1.3 - 0.7 = 0.6). Consequently, the distance from the center of rotation to the nodal point is 0.6 cm. For the image plane distance $d_y$, one option would be 3.8 cm because the center of rotation is 1.3 cm from the cornea[24,31] and the recommended eye relief (distance from cornea to display surface) is 2.5 cm[29]. If no lens is used, the physical display surface is identical to the image plane, so they are located at the same distance. Naturally, the observer should not be expected to accommodate to a near distance of 3.8 cm. A more appropriate value is 25 cm to represent near working conditions in which an observer manually manipulates stereoscopic objects. For this analysis, $d_y$ was 25 cm.

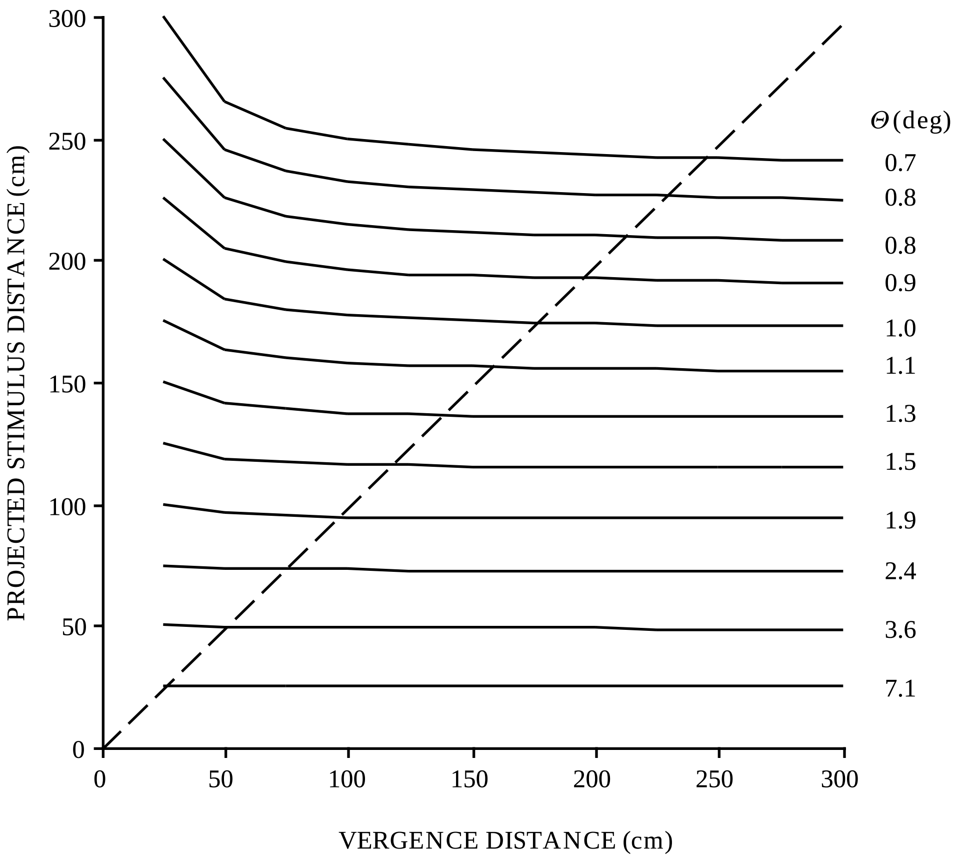


*Figure 4. Stimulus distance versus vergence distance for different horizontal positions of the stimulus on the display. The stimulus is fused along the dotted line. The angle Θ is the angle between the viewing normal and the line from the center of rotation to the stimulus position on the display, as shown in Figure 3. The vergence distance is shown by points $f_1$ and $f_2$, and stimulus distance is shown by points $s_1$ and $s_2$ in Figure 3.*

An example of the quantitative analysis can be illustrated for the condition in which the observer is initially fixating at a distance of 25 cm (*x*-axis), and the stimulus is displayed such that it is projected binocularly in space at a distance of 300 cm (*y*–axis). To fuse the target, the observer must diverge. As the observer diverges, the projected distance to the stimulus shifts inward. By the time the observer has diverged to 242 cm, the stimulus is perfected fused. Consequently, the projected position of the stimulus shifts inward from 300 cm to 242 cm during the divergence movement. If the observer continues to diverge to 300 cm, the projected stimulus distance shifts inward to 241 cm. Stimuli that are projected to be close (such as 50 cm) have little variation as the observer changes vergence. The greatest change occurs in the upper left corner of the figure, in which the observer has a near vergence distance (such as 25 to 100 cm) while the target is projected to be far away (such as 300 cm). If the observer is initially fixated at the projected position of the stimulus (along the dashed line), then no vergence changes are required to binocularly fixate the target.

The image plane distance can be designed such that the observer must accommodate at any distance. In the previous analysis, the image plane distance was assumed to be 25 cm. Figure 5 shows the effects of vergence on the distance to binocular projected virtual object when the image plane distance is set to distances of 25, 100, 200, and 300 cm. As shown in this figure, the distortion decreases as the image plane distance increases. For example, the projected distance to the binocular stimulus remains approximately constant with different vergence distances if the image plane distance is set to 300 cm.

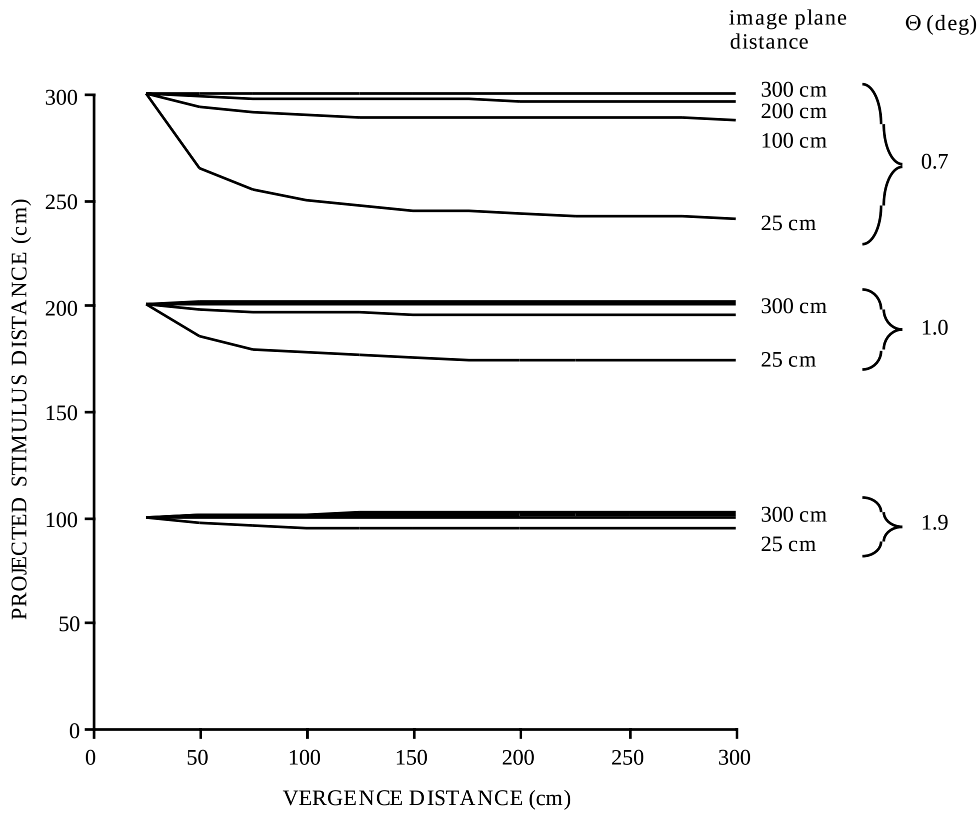


*Figure 5. Stimulus distance versus vergence distance for different vertical distances to the display surface.*

## DISCUSSION

The projected distance to a binocular virtual object varies with ocular vergence. The greatest variation occurs when the image plane distance is relatively close, such as 25 cm. Naturally, one could reduce these effects by designing the display such that the image plane distance is far away, such as 300 cm. However, if the observer is expected to view a binocular virtual object that is stereoscopically presented up close, then an inherent accommodation-vergence conflict will be produced by the accommodative demand that is set to the far condition.

The shift in the projected position of the virtual target with vergence is similar to the instabilities that can occur in head–tracked HMDs. If a virtual object in a HMD is supposed to be appear stationary in the world, the target must be displayed according to head position. However, time delay between the observer's motion and the image update causes instabilities in the depicted position of the virtual object[32]. The effects of this phenomenon were investigated in a recent study by McCandless, Ellis, and Adelstein[33], in which observers aligned a pointer with the apparent position of the virtual object while translating from side–to–side. In that experiment, the position and orientation of the virtual object on the display were not precisely synchronized with head position due to a time delay. Although the 31 ms time delay produced perceptual instabilities, it did not completely eliminate the ability to localize the virtual object.

Displaying a virtual object according to eye position represents a useful means of reducing perceptual distortions of virtual objects. A related distortion can be caused by a conflict between the vergence and accommodative demand. This distortion can produce disruptions in binocular stability as well as symptomatic complaints[34]. One means of reducing this conflict is to adjust the accommodative demand as the observer changes vergence. This technique was implemented in a system with a movable screen in a head mounted binocular viewer[35]. In a related design, accommodative demand to a virtual image was adjusted with a relay-lens[36]. The virtual image could shift from 20 cm to 10 cm in less than 300 ms by moving the relay-lens approximately 4 mm.

The projected shift in the position of binocular virtual object could have adverse effects in critical HMD applications, such as surgery or teleoperation in which fine visuo–motor control is required. Consequently, a head–mounted display that depicts a stationary projected virtual object should present the object on the display according to eye position information.

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