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*Ames Research Center*
*Moffett Field, California*
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Michael Wallace McGreevy and Stephen R. Ellis
Ames Research Center

The design and implementation of a perspective display of air traffic for the cockpit is discussed. Parameters of the perspective are variable and interactive so that the appearance of the projected image can be widely varied. This approach makes allowances for exploration of perspective parameters and their interactions. The display was initially used to study the causes of horizontal maneuver biases found in experiments involving a plan view air traffic display format. Experiments to determine the effect of perspective geometry on spatial judgments have evolved from the display program.

The display depicts a volume of visible airspace around “own ship,” the space around the aircraft symbol representing the pilot’s own aircraft so that separation may be monitored in all directions. Symbols and metric aids which enhance the sense of space derived from perspective and which show both horizontal and vertical separation in an integrated format are contained in the display. By use of switches, the data tablet, and the computer terminal, the display may be altered in content, in direction of view, and in geometry of projection.

Perspective is one way that three-dimensional (3-D) information may be projected onto a two-dimensional (2-D) surface. Geometry of perspective can be described conveniently as a frustum whose position, orientation, and shape define the appearance of the projected image. The frustum shape embodies the bundle of light rays that produce a rectangular image. By defining the perspective algebraically as it is relative to the position of own ship, rather than as it is relative to the display “window,” the visible space around own ship becomes part of the definition of the perspective. Several scaling techniques and other adjustments to the perspective are used to tailor the geometry for effective presentation of three-dimensional traffic situations.

INTRODUCTION

A perspective display of air traffic for the cockpit (fig. 1) that was designed and implemented at the Aerospace Human Factors Research Division at NASA Ames Research Center (McGreevy, 1982; McGreevy, 1983) is described. The perspective display format is a further development of the Cockpit Display of Traffic Information (CDTI) format studies conducted at Ames as part of NASA's research program in human-machine interaction. Use of perspective has raised an important set of questions about spatial information displays in general.

The display was created on a vector graphics device that allows dynamic displays. Use of the dynamic capability of the display device includes presentation of dynamic elements; i.e., air traffic, and also allows dynamic interaction with the viewing geometry; i.e., the position of the eyepoint and the perspective projection parameters. The appearance of the displayed scene can be varied continuously between that of a “game-board world,” and that of a vast space with a distant horizon. The wide variability of the apparent spatial relationships raises some interesting questions about the use of various perspective parameter values in spatial displays, and in the accuracy of the spatial information derived as a function of the perspective geometry.

Several scaling techniques and metric aids are used to enhance the effectiveness of the perspective display. Since horizontal and vertical dimensions of interest in aviation are dissimilar in magnitude, the altitude dimension of the airspace is scaled. This scaling is calculated relative to the perspective geometry to provide a consistently scaled image, regardless of perspective changes which change the volume of the airspace displayed. The aircraft symbols are scaled so that own ship remains constant in screen size, independent of perspective changes, whereas the other aircraft symbols vary in size according to their distance from own ship and the “strength” of the perspective. Metric aids such as lines that connect three-dimensional (3-D) points to their “map” locations, and tic marks that indicate relative altitude, supplement the perspective sense of volume so that metric spatial judgments are also possible. For the most effective representation of the traffic surrounding own ship, the eyepoint is removed from own ship so that the view around own ship, including below and behind it, is presented.

PLANVIEW DISPLAYS

Traffic displays have typically been planviews, where the screen face represents a segment of a map, and altitude is collapsed and encoded (Boeing, 1977; Jones, Schrader, and Marshall, 1950; Palmer, Jago, Baty, and O’Connor, 1980; and Weiss and Bush, 1972). Altitudes of the displayed aircraft have been represented in a variety of ways, most
notably by associating a data tag with each aircraft symbol, in which a number represents altitude. Relative altitudes must then be derived from mental arithmetic, or from rough magnitude comparisons.

Another planview encoding scheme considers the altitude dimension to have a coarse resolution that is encoded by a few colors or shapes (Bird, 1975; Hart, 1981). For example, in figure 2 a narrow altitude band is defined as "at own-ship's altitude" and aircraft within this band are represented by a hexagon. An aircraft that is above own-ship's altitude is represented by the top half of a hexagon, and an aircraft that is below is drawn as a bottom half. A pseudo-spatial planview format proposed by Bird (1978) encodes altitude by line length. These schemes may require excessive time and effort for the pilot to visualize the dynamic spatial relationships of displayed aircraft; however, other methods may be easier to interpret.

AN ALTERNATIVE REPRESENTATION

The design of the perspective traffic display was motivated by the desire to explore traffic displays that present altitude information in a more integrated format. A perspective display can present the same information as a planview, but in a different format, one that does not collapse the vertical dimension. For example, the encounter situations in figure 1 and figure 2 are identical and only the display method differs. Researchers found that pilots have a pronounced bias toward horizontal maneuvers when basing these maneuvers on planview information (Smith, Ellis, and Lee, 1984). Although there are several plausible explanations within the context of aircraft performance, passenger comfort, and ATC rules, the researchers wanted to know to what extent the planview representation is itself responsible. Experiments were conducted in which maneuver decisions were made by pilots using both the perspective display and a planview display. Experimental results (Ellis, McGreevy, and Hitchcock, 1984) indicate that maneuvers involve significantly greater vertical components when the perspective display is used (fig. 3). This appears to be due to the fact that spatial information is presented in a more homogeneous manner in the perspective display. The research also indicates that when using the perspective display, pilots are more often able to maintain standard separation (>1000 ft vertical, >3 n. mi. horizontal) from intruding aircraft than when using the planview display of the same encounters. Clearly, the format used for presenting the spatial relationships among the aircraft is an important consideration.

Several questions are raised by the proposed use of perspective situation displays in the cockpit. It is important to know the accuracy of the pilots' judgments of the spatial relationships among the objects in the airspace when this information is derived from perspective displays. This is particularly important when the spatial interpretation must be derived with respect to a point or object within the scene which represents one's own position. Of primary importance in a traffic situation display is the effective communication of the direction of the intruder, i.e., azimuth and elevation relative to one's own-ship's heading and altitude, as well as range of the ship. The variation of judgment accuracy as a function of the perspective parameters must be known in order to design displays which best represent the spatial relationships of interest to the pilot.

Research to answer these questions has begun. Initial results (McGreevy and Ellis, 1984) indicate that the field-of-view angle has a systematic and significant effect on direction judgments. This research was made possible by the flexible and interactive nature of the program described in this report, and by the exploration of perspective that this program made possible.

DESCRIPTION OF THE DISPLAY AND ITS USE

Hardware and Software

The display program is written in OMSI Pascal and runs on a PDP-11/70 minicomputer under the RSX-11M operating system. The display device is an Evans and Sutherland Picture System 2 (PS-2), which provides Fortran subroutines that are called from the Pascal program. Interaction with the program is via a Digital (DEC) VT-100 video display terminal, PS-2 function switches (toggle and momentary action), and data tablet. Hardcopies may be made on a Versatec electrostatic printer/plotter.1

Symbols and Metric Aids

The symbols and metric aids that appear in the display are designed to make clear the current and future spatial relationships among own ship and surrounding traffic. The use of metric aids is especially important since perspective images without metric aids are full of ambiguities. This is especially true of highly abstract images like those used in this display, which have few of the cues commonly used to interpret spatial relationships in real-world scenes. Since our

1Most of the display examples in this paper were done on the Versatec. Line qualities of the actual PS-2 display and the hardcopies differ in several ways. Vectors drawn by the Versatec are black on white, while vectors in the actual display are drawn in a range of intensities on a dark background. The judicious use of symbol intensities and intensity gradients is useful on the actual display for reducing visual clutter, but our hardcopy device does not support this necessary feature. Also, the vertical alignment of dashed lines in the hardcopies can cause some effects not present in the actual display.
Evans and Sutherland Picture System 2 can provide dynamic calligraphic images, but not color, shading, or polygon fill, all elements of the display are comprised of monochromatic lines.

If only the aircraft are displayed, their relative positions are very difficult to interpret (fig. 4). Whether the aircraft are pointed into or out of the screen is unclear. Some manipulation of the perspective can enhance the size constancy cue and emphasize the diminution of size with depth, but this is of limited value when used alone. Moving the eyepoint provides motion parallax, and this is also helpful, but not sufficient.

The addition of a textured ground plane provides a very definite improvement in the sense of depth (fig. 5). A regular grid can be drawn efficiently with straight lines and is a useful aid for measuring. The orientation of the grid has a distinct effect on the apparent depth: when grid lines are parallel to the top and bottom of the screen, the effect is diminished (Gillam, 1971; Hering, 1868). Also, use of a ground plane for depth effect encourages use of a view that includes the grid seen at some slant angle. The relationship between each aircraft and the grid is unclear, however, unless more symbology is added.

Some thought was given to putting shadows beneath the aircraft, but a line connecting each aircraft to its map position on the grid is more explicit (fig. 6). The regular lines of the grid can be used to measure the horizontal separation between map positions. These metric lines also provide information about vertical position relative to one’s own-ship’s altitude. However, since size is diminished with depth in perspective images, the length of these lines cannot be compared to judge relative altitude. For this reason, each line is marked at the point where it intersects own-ship’s altitude. If own ship is above the intruder, the intruder’s line is extended up to own-ship’s altitude. Tic marks at thousand foot increments on the metric lines provide a measurement of the relative vertical separation (fig. 7). Additional symbology for representing predicted future map position and altitude is designed with similar considerations (fig. 8).

The use of metric aids supplements perspective by clarifying the magnitudes of separation. Thus, a perspective display can provide an integrated and precise sense of space, which does not require encoding of horizontal or vertical separation. This approach is consistent with the idea that displays should emphasize the parameters of interest, not just the objects of interest (Fulzon, 1982). More detail about the symbols and metric aids follows.

**Aircraft position symbols**—These indicate the 3D position of an aircraft. The exact position is at the nose of the aircraft symbol. The trend of a turn may be indicated by the bank of the wings, and vertical trends may be indicated by the pitch of the aircraft. Symbol shapes are aircraft-like and can represent different types of aircraft so that expectations regarding behavior may be formed. Aircraft symbols are filled in for better visibility. Aircraft symbol definitions (fig. 9) are stored in separate files.

**Horizontal reference grid**—This reference plane has grid lines that are separated by an integer number of nautical miles, typically 1 to 3. The grid is positioned at a selectable number of feet below own ship. A typical default is 5000 ft. Aircraft below the grid are not displayed. The grid is aligned with own-ship’s heading to provide a convenient reference for lateral and longitudinal separation. Grid lines which are perpendicular to the heading move below own ship, in the opposite direction from own-ship’s heading, at a rate equal to own-ship’s speed. This maintains a stationary reference, gives own ship a natural appearance of motion, and conveys a sense of appropriate relative motion among aircraft, even though own ship is fixed at the center of the screen. The intensity of the grid is greatest near the eye and decreases progressively toward the back of the presented airspace. This aids depth perception and eliminates the clutter near the horizon (which is a problem in the hard-copies, which have only one available line intensity) caused by the more closely spaced grid lines (because of perspective).

**Nose lines (metric lines)**—A nose line connects an aircraft’s position in three dimensions to the projection of that position on the reference grid. This gives the horizontal (map) position of the aircraft. The nose line is also called a metric line since it carries all the altitude metrics.

**Own-ship altitude mark**—This mark is an “X” that intersects an aircraft’s nose line at the point where it intersects own-ship’s altitude plane. If the intruder is below own-ship’s altitude, the noseline is extended up to own-ship’s altitude, where the “X” is drawn. It provides a vertical separation reference for each intruding aircraft relative to own ship.

**Thousand foot tic marks**—These indicate thousand foot intervals along the nose line between the “X” at own ship’s altitude, and the altitude of the intruding aircraft. Whole thousand foot intervals begin at the own ship altitude mark, and fractions of this interval are located near the intruder. Tics are brighter than the nose line so that they are prominent.

**Predictor of future position**—Predictors are presented in 3D so that they also show predicted future altitude, in addition to showing horizontal position. Predictor lines...
extend from the nose of each aircraft symbol (exact present position) to the future position. Predictors may be curved, if the appropriate data are available. Position predictions are typically 60 sec into the future. It may be useful to selectively present predicted position so that all aircraft are displayed, but only potential threats are displayed with predictors.

Future position metric lines—These lines are similar to nose lines, but are placed at the “future” end of the predictor. They also help to clarify the current direction of travel, which is otherwise subject to ambiguity. Some differentiation among the metric lines is done by using various intensities. One recent addition has been to make the current position and metrics bright while those representing the future are progressively dimmer.

History dots—These mark the recent path of an aircraft at regular time intervals. They are a more reliable indicator of future position than predictors when position sensor noise is present.

Data tags—Tags contain alphanumeric information. The content may vary from simply showing the flight number, to including horizontal and vertical speeds. Tags in the development version of the program label the intruder aircraft by number. Tags used in the display program for experiments are quite variable in content and can be completely suppressed, if desired.

Tag lines—These lines are used to connect aircraft symbols and their tags. These are often unnecessary, except in very crowded skies or very compressed (because of extreme perspective) views.

Dynamics

The program has a primitive trajectory generator which has the capability to move all of the aircraft symbols. Its purpose was to provide dynamics during the early development of the display. Once a file of initial positions, headings, and rates is input, motions of the symbols are generated for as long as they are needed. The file format and an example are shown in figure 10. Trajectories may be straight or curved and they may have selectable vertical speeds. Curvature depends on the initial turn rate of the symbols. The dynamics may be stopped and restarted at any time so that different viewing geometries may be assessed in different situations.

The experiment version of the program can also be driven by any aerodynamics model, traffic generator, or traffic data. The display has been used to present specific encounters in numerous experiments. It has also been used to visualize terminal traffic, using data obtained from FAA radar. In a recent experiment, the display was installed in a Boeing-747 simulator. As the pilot maneuvered to avoid intruding aircraft, the own-ship symbol would bank, pitch, and move in correspondence with the control inputs.

Tag Placement Algorithm

The position of each tag within the 3-D scene is solved in two dimensions on the plane of projection by a priority placement algorithm which ensures that the tag is in a readable (nonoverlapping) position. The algorithm considers the projected positions of all aircraft symbols, metric lines, and other data tags. It then selects the best free-screen position located near the appropriate aircraft symbol, based on a priority scheme. The priorities rank the preferred positions of the tag relative to the aircraft symbol, and may be changed interactively. Tags are drawn with bright vectors and are distinct against the background on the vector display.

A further refinement of the tag algorithm reduces the distracting jumping of tags from one position to another. The Tag is made to glide between positions. However, the sudden onset of motion is still slightly distracting. Ideally, the tag should smoothly accelerate to the glide velocity and smoothly decelerate. This may be very important in reducing visual distractions in the cockpit.

Interactive Features

The geometry of the perspective projection can have a dramatic effect on the apparent spatial relationships of the objects displayed. In figure 11, for example, the same traffic situation appears very different with different perspectives. So that the different perspective effects can be explored, the display program has a “synthetic camera” with which to view the synthetic airspace in real time. Variation of the field-of-view angle is similar to changing lenses, or zooming between wide-angle and telephoto settings. Changing the camera-to-subject distance at a given “lens” setting changes the amount of space that is visible around the subject. Appropriate changes in both the camera-to-subject distance and the field-of-view angle allow the apparent depth of background objects to be varied somewhat independently of the amount of space surrounding the subject in the picture frame. The synthetic camera can also create images which cannot be made with a real camera, by scaling objects or dimensions selectively as a function of perspective. Such effects on the images are easily explored because the perspective parameters can be controlled dynamically through the use of a continuous range of values.

The display program is designed to allow experimenters to view dynamic encounter scenarios from all possible
viewpoints, with all possible lens settings. The viewpoint is manipulated in real time by moving a stylus over a data pad. Field-of-view angle may be manipulated in the same way. As an encounter progresses, the perspective may be varied, even as the aircraft move. This allows a great deal of freedom to explore the effects of different perspective views. Because the display is so flexible and interactive, its use in display design minimizes the number of iterations through the cycle of specification and programming. It also provides immediate feedback when varying the perspective parameters so that the contributions of each one to our particular needs to be assessed.

It is useful to be able to monitor the numerical values of the perspective parameters as they are interactively varied or to set them to specific values. Correlating these numerical values with the viewed effects is essential for understanding the relationships involved. Whereas some analyses of pictorial geometry are limited to the appearances of static photographs, this program provides the complete numerical specification of its perspective for each picture. This includes the perspective parameters, the homogeneous transformation matrices, object coordinates, and aircraft and altitude scale factors.

The essential parameters are output at the bottom of the graphics display (see fig. 4, for example), while the others may be displayed on the alphanumeric terminal screen. Essential parameters include the field-of-view angle, elevation of the viewpoint above own-ship's altitude plane, azimuth of the viewpoint relative to own-ship's heading, the distances between the reference point (own ship) and the clipping planes, the distances between the station point (geometric eyepoint) and the reference point, and the dimensions of the window in the picture plane. These are more fully explained in the section "Basic Display Geometry."

**View Selection Switches**

After a "viewing geometry" of interest has been found by using the data pad to interact directly with the perspective, it is not necessary to remanipulate the pad to return to the view. Three of the function switches are used to set up any of eight predefined views. When a particular viewing geometry is to be saved, it is defined as a member of a set of eight view selections that can be searched by name. In order for this to be done, the essential parameter values, which are displayed (along with other information) at the bottom of the screen, must be entered into a file. This process allows all of the parameters of the view to be set with a single switch. It also makes it possible to group similar kinds of views together for experiments or for demonstrations. The view-switch file format and an example are given in figure 12.

Any number of view sets may be defined and stored as files for later use. Using predefined views, it is possible to switch directly from view to view. Any of the views may serve as a starting point for further manipulations which use the data pad. View parameters can also be entered into a file without having to use the pad at all.

**Pad Mode Switches**

Pad mode switches serve to map perspective variables to the data pad for interactive manipulation. Using the pad, the position of the eye point is variable in real time. The surface of a hemisphere of user-selected radius is mapped onto the data pad, which allows movement of the eye over this surface by moving the stylus over the pad. Since own ship is always the point of regard, this is equivalent to swiveling the frustum of vision about the pivot point of own-ship's position.

The data pad is also used to vary the perspective geometry in real time. This is equivalent to varying the shape of the frustum of vision (see "Perspective Geometry" section). By selecting appropriate modes with the function switches, it is possible to hold one of the basic perspective parameters constant while manipulating the other two by using the data pad. Those basic parameters are the field-of-view angle, the eye-to-ownship distance, and distance from ownship to the nearest clipping plane. These are discussed in detail in the perspective geometry section.

Thus, the viewing geometry of the eye point and the perspective is easy to manipulate, allowing the spatial relationships in the display to be arranged as desired. This is important because the spatial information in the display must be accurately perceived, and the effects of the various perspective parameters must be well understood.

**Other Switches**

A snapshot switch allows hardcopies to be made as the display program runs. Since the essential viewing geometry parameter values may be displayed on the screen by switch selection, hardcopies may record both the appearance and the underlying geometry of any display. Several switches allow elements of the display to be erased.

**Keyboard Interaction Mode**

The keyboard interaction mode is entered by use of a function switch and remains active until explicitly exited at the keyboard. This mode causes the program to pause. Since changes made in this mode can alter the appearance of the display, a single display loop may be executed for observation of effects while the program is in the keyboard mode.
This mode of interaction allows: direct numerical specification of all parameters of the viewing geometry; the output of the matrices that embody the geometric transformations; the output of the 3-D coordinates of each aircraft; the adjustment of the brightness of elements in the display; the change of the symbol scale; the selection of traces and dumps. The keyboard interaction mode also allows redefinition of the following: the viewing geometry selection switches; the grid distances, the orientation and motion; the aircraft initial conditions and trajectories; and the tag placement priorities.

BASIC DISPLAY GEOMETRY

Projections

There are many ways to project three-dimensional information onto a 2-D screen. The planar geometric projections are categorized in figure 13. Both the parallel and the perspective projections were considered for our display.

In a planar projection from 3-D to 2-D, a projector from each transformed point intersects the picture plane. In both parallel and perspective projections, all projectors pass through a single point, called the center of projection, or eye point. In the parallel case, that point is infinitely far from the object which results in projectors which are parallel (fig. 14(a)). In the perspective case, the point is at a finite distance and the projectors converge (fig. 14(b)). Note that as the center of projection approaches large finite distances from the object, the projectors appear increasingly parallel. Thus, parallel projection is an extreme limit of perspective (fig. 14).

A Variety of Parallel Projections

In orthographic parallel projections, the projectors are all parallel to the viewing axis (which is orthogonal to the picture plane). In orthographic top/front/side views, the viewing axis is also parallel with one of the principle axes of the object or scene (fig. 15). The top/front/side view method aligns two of the three principle axes with the picture plane axes and collapses the third (along the viewing axis). The generic plan-view display is an orthographic top view of the world around own ship. The viewing axis is parallel with the vertical (or altitude) axis, one of the principle axes. Alternative altitude displays have used the orthographic front or side views.

Whereas top/front/side views are restricted to views along one of the principle axes, axonometric projections allow views of the scene or object from different directions. Isometric views foreshorten all axes equally so that the scale on each axis is the same. Dimetric views shorten two axes equally. Trimetric views have all three axes scaled differently. Whereas axonometric projections provide views with some improved sense of space, objects at various distances do not appear to be appropriately scaled relative to one another. For example, two cubes of the same size, located at different distances from the picture plane, can appear to be different sizes or shapes. The expectation of reduction of apparent size with depth seems to be responsible for the illusion.

The oblique projection differs from the orthographic in that the projectors are not parallel with the viewing axis. This allows the front face of a boxy object to be undistorted while showing some of the sides and top or bottom. Since there is no advantage to be gained for the display of the airspace, it was not considered.

Types of Perspective

Perspective can be categorized as one-, two-, or three-point perspective (fig. 16). This breakdown is based on two-dimensional construction techniques used in draughting and is of limited use in spatial display specification. It is included for completeness. Perspective is analyzed intuitively, geometrically, and algebraically in the next section.

With one-point perspective (fig. 16(a)), the principle view vector (viewing axis) must be parallel to one of the principle scene axes. In other words, only one scene axis pierces the picture plane. Convergence only occurs along that axis.

Two-point perspective requires the principle view vector to be perpendicular to one of the three principle scene axes (fig. 16(b)). No convergence occurs along the perpendicular scene axis. The other two axes pierce the picture plane, and convergence only occurs along these axes.

With three-point perspective (fig. 16(c)), the principle view vector is not parallel to any of the principle scene axes. All three scene axes pierce the picture plane, and convergence occurs along all three axes. This is the least restrictive category and corresponds well to the typical perspective of scenes in the real world. The display described in this report is capable of creating images in any of the three categories.

Other Types of Projections

The above discussed methods do not exhaust the possibilities for projections from three to two dimensions.

3In all discussion of perspective in this paper, the principle view vector is defined to be orthogonal to the picture plane. Thus, constraints on the view vector constrain the picture plane. It should be noted that, in general, the principle view vector need not pass through the center of interest.
Nonplanar projections and projectors which are nonlinear can be used for a variety of special purposes. For example, in a front- or side-view orthogonal projection of the airspace, a nonlinear scaling of the vertical and/or horizontal dimension could provide for better local resolution and still include the desired range (fig. 17).

**PERSPECTIVE GEOMETRY**

The mathematical abstraction of geometric perspective is a model of the physical geometric relationships of light rays, objects, and the eye of the observer. The simple model used here considers only the positions of key points in three dimensions and the projection of those positions into two dimensions. A geometric convenience which approximates the three-dimensional shape of the visual field, the frustum of vision, can be derived from the basic physical relationships involved in this projection.

**From Scene to Frustum**

The light reflected at each point in a scene propagates out in all directions according to a function based on the nature of the light source, object surface, their relative orientations, and any intermediate medium (e.g., air). A direction vector will suffice to model each of the rays that we wish to consider. (No color, intensity, refractability, or the like are retained.) If an eye is situated in this world, some rays will intersect that eye, though not all of them will come from a direction which allows them to enter the eye (fig. 18).

What is needed is a geometric approximation of the visual field of view. This can then be used to distinguish visible rays from invisible ones. As a consequence, it can be used to decide which parts of objects and scenes are visible, and which are not (fig. 19).

Let a transparent plane-segment \(P\), a rectangular piece of window glass be a good approximation, of length \(L\) and width \(W\) be placed a distance \(D\) from the eye, orthogonal to the principle view vector of the eye (fig. 20). Only the rays which pass through the plane and reach the eye will be considered to be visible. \(P\) is an image plane since it intersects the rays converging on the eye.\(^5\)

Consider the rays which pass through an edge of the plane \(P\) and converge at the eyepoint (fig. 21). These rays define a bounding plane that separates visible rays from invisible rays. An object which passes through any of the four bounding planes is clipped into a visible and an invisible segment (refer back to fig. 19), which lends the name “clipping planes” to these bounding planes. Together, the four planes, one at each edge of the plane \(P\), passing through the eyepoint, carve out a pyramid-shaped region of visible space, surrounded by invisible space. The eyepoint is at the apex of this pyramid and the base is at infinity (fig. 22).

Two more clipping planes, the hither and yon planes, can be added to complete the set (fig. 23). The hither plane is placed between the eye and the region to be viewed, and is orthogonal to the principle viewing ray. Visible space begins on the far side of this plane. This can be thought of as the glass of the display face. Nothing is displayed on the near side of it. The spatial world displayed begins on the other side of the hither plane. The yon plane can be placed on the far side of the region to be viewed. It, too, is orthogonal to the principle viewing ray. This plane removes from view those items which are beyond it.

Taken together, the six planes described carve out a truncated pyramid (frustum) of visible space. Everything within the frustum is projected onto the picture plane-segment, \(P\), which can be thought of as identical with the hither plane. The projected image is scaled and mapped onto the display screen.

**Algebraic Definitions of Perspective**

So far, the frustum that embodies the perspective transform has been treated as a geometric shape, but to conveniently manipulate it, an algebraic description is needed. First, define a set of axes such that the origin is at the eye point, the \(z\) axis is the central view ray, the \(y\) axis points up, relative to the eye point, and the \(x\) axis points right, relative to the eye point. This “left-handed” system is called the eye coordinate system (fig. 24).

It is useful to define two field-of-view (fov) angles, to help describe the shape of the frustum of visible space. The angle between the top and bottom clipping planes is the vertical field-of-view angle, \(\text{fov}(\gamma)\). The angle between the left and right clipping planes is the horizontal field-of-view angle, \(\text{fov}(\chi)\) (fig. 25).

\(^{4}\)It is one simple component of a more life-like model. Models of the characteristics of light, objects and observers can range from extremely simplified models to those which include the detailed nature of the interaction of light and matter, and more complete optical systems.

\(^{5}\)If every quality of the rays that pass through \(P\) could instead be generated at \(P\), the viewer would theoretically be unable to tell the difference. A goal of the perspective transform approach to spatial information transfer is to present enough convincing features on the surface of the display screen, features that imply a voluminous world with physical depth, so that the viewer forgets about the surface plane and takes as self-evident the spatial world “beyond” the screen.
Window-Based Definition

In the development of the frustum from the scene, a window-like plane-segment \( P \) was defined. Recall that \( P \) has width \( W \) and length \( L \), and is a distance \( D \) from the eye. The ratios \( D/(W/2) \) and \( D/(L/2) \) define the shape of the frustum (fig. 26). Note that

\[
\text{cotangent}[(1/2)\text{fov}(x)] = D/(W/2)
\]

and

\[
\text{cotangent}[(1/2)\text{fov}(y)] = D/(L/2)
\]

Thus, either \((D,W)\) or \(\text{fov}(x)\) defines the horizontal shape of the frustum and either \((D,L)\) or \(\text{fov}(y)\) defines its vertical shape. If \(W = L\), the aspect ratio of the view is square and \(\text{fov}(x) = \text{fov}(y)\). So that

\[
\text{cotangent}((1/2)\text{fov}) = D/(W/2)
\]

Defining the perspective in terms of the distances \( D, W, \) and \( L \), results in a window-based definition.

Reference-Point Definition

An alternative formulation relates the shape of the frustum to a point within the visible region, which will be called a reference point. In the display of the airspace, this is the position of own ship. For the purposes of defining the visible space around own ship, this alternative formulation is more useful than is the window-based definition.

What is needed is an equation for \( \text{cotangent} \left( \frac{\text{fov}}{2} \right) \) that is in terms of more useful distances than \( D, W, \) or \( L \), which were used above. These distances are the eye-to-eye reference point distance \( (E) \), and the minimum distance from the reference point to the clipping planes \((R)\) (fig. 27). Confining the reference point to the principle view ray ensures that the minimum distances between the reference point and the left and right planes are equal:

\[
R_1(x) = R_2(x) = R(x)
\]

The distances between the reference point and the top and bottom planes are also equal:

\[
R_1(y) = R_2(y) = R(y)
\]

Further, since there is only one reference point for both horizontal and vertical planes, and assuming that

\[
\text{fov}(y) = \text{fov}(x) = \text{fov}
\]

then

\[
R(x) = R(y) = R
\]

Using \( E \) and \( R \), the desired expression is

\[
\text{cotangent}[(1/2)\text{fov}/2] = (E/R) \cdot \text{cosine}([\text{fov}/2])
\]

Any two of \((\text{fov}, E, R)\) define the shape of the pyramid of visible space surrounding the reference point, and thus define the perspective of the view:

\[
\text{fov} = 2 \cdot \arcsine(R/E)
\]

\[
E = R/\sin(\text{fov}/2)
\]

\[
R = E \cdot \sin(\text{fov}/2)
\]

To define the frustum of visible space, the hither and yon distances must be defined. Their positions are largely arbitrary. It is useful to do without the yon plane altogether. The hither plane is positioned a distance \( R \) from the reference point (own ship in the airspace display), so that all clipping planes are the same distance from own ship.

Algebra of Projection

Now that the perspective information (frustum shape) is contained in the cotangent ratio, the algebra of projection of a point from 3-D to 2-D can be defined. The purpose of the following description of the algorithm is to show the algebraic relationship between the frustum shape and the perspective projection. In practice, the computations differ for the sake of efficiency. See Newman and Sproull (1979) and also Foley and Van Dam (1982) for information about the matrix algebra used in computer graphics.

Figure 28 illustrates the geometry of perspective projection. The point \( P(x,y,z) \) is a distance \( D \) from the eye point, along the \( z \)-axis. The vertical dimension of the diagram represents the \( y \) dimension. Assume \( x = 0 \) so that the point \( P \) is in the plane of the diagram, for simplicity. To find the projection of \( P(x,y,z) \) onto the picture plane, the point \( P'(x',y') \), observe (by similar triangles) that

\[
y'/d = y/D
\]

so that

\[
y' = d \cdot y/D = d \cdot \text{tangent}(\alpha)
\]

where \( \alpha \) is the angle between the \( z \) axis and the projector for \( P(x,y,z) \). (For \( x \neq 0 \), \( \alpha \) is the angle between the \( z \) axis and the projection of the projector onto the \( yz \) plane.)
To obtain the scaling of \( y' \) which takes into account the perspective (frustum shape) we have defined, \( y' \) must be scaled by \( \cot(\text{fov}/2) \):

\[
y'' = y' \cdot \cot((1/2)\text{fov})
\]

\[
y = d \cdot \tan(\alpha) \cdot \cot((1/2)\text{fov})
\]

(When \( x \neq 0 \), a similar calculation determines \( x' \) and \( x'' \).)

Points \( P''(x'', y'') \) with \(-d \leq x'' \leq d\) and \(-d \leq y'' \leq d\) are within the visible part of the picture plane. Points outside these ranges are not visible. This can be visualized by imagining all visible points \( P \) as being painted on the hither plane (or plane segment \( P \)). Visible points are then scaled according to the screen resolution, \( y\text{-resolution} \), of the display device to obtain the screen coordinates, \( y\text{-screen} \). For example,

\[
y\text{-screen} = y'' \cdot (1/2)y\text{-resolution}/d
\]

**Examples of Projection**

Figures 29, 30, and 31 illustrate the projection of an object using perspective. Figure 29 shows the frustum with the object, a cube, at the origin and at the reference point. The cube is defined in terms of the origin. It is then translated to the reference point. Prior to translation, the cube may also be rotated and scaled. The effect of this is illustrated in figure 30, along with the projection of the cube onto the picture plane. Figure 31 shows what the observer sees if his eye is at the eye point.

Figures 32, 33, and 34 illustrate a clipped object, and its relationship to the frustum and the screen. Figure 32 is similar to figure 30, except that the cube has been translated so that it is partially outside the visible region. Figure 33 is a top view of the frustum. It clearly shows which part of the cube is outside the visible region within the frustum, and the corresponding projection. Figure 34 shows what the observer sees if his eye is at the eye point.

**ADJUSTMENTS TO THE PERSPECTIVE**

In addition to the standard perspective effects, the display technique includes adjustments, constraints, and rescaling which refine the display geometry. The perspective of a scene is embodied in the frustum: in order to shape the desired view, it is necessary to orient and shape the frustum. The frustum of visible airspace that results from the use of perspective is not a conveniently shaped volume. Typical traffic separation criteria are defined in terms of a cylindrical volume around an aircraft. The difference between these two shapes was at first thought to be a problem. In looking at various alternative solutions, it became clear that the frustum shape could be used directly, if it were slightly modified.

To carve out a particular volume of airspace, the frustum must be oriented, shaped, and partitioned in a particular way. Constraining the orientations and shapes constrains the perspective. Unless additional clipping is done, the entire volume within the frustum is visible. Unless other scaling is done, the relative sizes of objects depend only upon the position and shape of the frustum. To obtain the desired view, then, constraints and additions must be applied to the frustum.

**Position and Orientation of the Viewing Frustum**

In general, the viewing frustum could be placed anywhere in the world/model. It could be free to move through the world/model (as the eye does in its gaze during the motion of the head and the body), with the direction of the frustum's central visual ray being independent of its direction of travel. It may instead be fixed with respect to the world/model, staring always in one direction. Another alternative is to allow the orientation of the viewing frustum to be established relative to a moving part of the world/model. This last option is used to constrain the orientation of view in this project: the viewing frustum is positioned and oriented relative to own ship.

For viewing the airspace surrounding own ship, the principle viewing vector (central visual ray) is made to intersect own-ship's position so that own ship will be at the center of the display (fig. 35). This ensures that own ship is surrounded by a buffer of visible space, regardless of the orientation of the eyepoint about own-ship's position.

The viewing vector is free to “tether” about the pivot point at own ship. To visualize this, picture the eyepoint as analogous to a tetherball, the principle view vector as analogous to the rope, and own-ship's position as analogous to the point of attachment of the rope to the pole. The orientation of the view vector can be described by its elevation above the horizontal and the azimuth from the own-ship's heading. No roll angle about the view vector was considered, since the display represents an objective view of the world, which contains own ship; it does not represent a window from own ship to the world. The length of the view vector (analogous to the rope length), in addition to its elevation and azimuth, specifies the eyepoint.

The viewing vector is further constrained to be at, or above, own-ship's altitude, but not below it. This ensures that the horizontal grid plane is adequately visible for use in making separation judgments. Thus, the eye point is confined to points within a hemisphere centered on own ship, whose flat side corresponds to own-ship's altitude.
Horizontal Clipping Planes

The frustum-shaped region of visible airspace is inconvenient, particularly if the principle view ray is not parallel to the ground, which is the case when the scene is viewed from other than own-ship's altitude. To simplify the entry and exit of aircraft into the scene, two horizontal clipping planes are established: one that slices through the frustum, and one that belongs to the frustum.

By requiring a simple relationship between the vertical field of view angle \( (\text{fov}(\psi)) \), and the angle between the principle view ray and the horizontal plane (elevation, \( e \)):

\[
\begin{align*}
e &= \frac{1}{2} \text{fov}(\psi)
\end{align*}
\]

the top clipping plane of the frustum can be made horizontal. By constraining the frustum in this way, it is assured that all aircraft that enter or exit the scene from above are doing so at the same altitude, regardless of the depth into the scene. Not all illustrations of the display have this constraint, but it is highly recommended in any practical application.

A horizontal grid plane, below own ship and parallel to the ground, is part of the scene. This defines a seventh clipping plane, in addition to the six that comprise the frustum. Aircraft below the horizontal reference plane (which is some selectable distance below own ship), are not displayed.

Hither Plane Position

Recall that in the reference point definition of perspective, the hither plane is not constrained, but that the distances from the reference point to the left, right, top, and bottom clipping planes are constrained (and are equal to range, \( R \)). By establishing the constraint that the distance between the reference point and the hither plane also equals \( R \), the local visible space around own ship is made symmetrical. This allows the rotation traces of the left, right, and hither planes to be identical for rotations about own-ship's vertical axis. In this way, aircraft beyond \( R \) in the direction of these planes are not displayed, and aircraft within a distance \( R \) are always displayed.\(^6\) This provides a horizontal distance (\( R \)) on three sides of own ship that is the minimum range displayed (fig. 36).

Altitude Scaling

Although the frustum is defined to be horizontally and vertically symmetrical, the separation standards in aviation are far from symmetrical. A 1000-ft vertical separation and 3-n. mi. horizontal separation is typical. This is an asymmetry factor of approximately \( (3*1852)/304.8 \), which indicates that the vertical dimension might be usefully scaled up by approximately a factor of 18. It is important to scale the horizontal and vertical so as to obtain the best resolution for each. Such a scaling may be a compromise between equal resolutions and natural appearance.

The actual scaling factor used depends upon the shape of the frustum and the distance between own ship and the horizontal reference plane. This ensures that the own-ship symbol is displayed at a constant distance above the reference plane, regardless of perspective, which provides an invariant vertical reference.

Recall that the principle view ray intersects the center of the screen and own-ship's position in 3-D, so that own-ship's position projects to the center of the screen. The scaling is defined so that for an elevation angle of zero, the point of intersection of own-ship's vertical axis and the horizontal reference plane is halfway between the center and the bottom of the screen (fig. 37). This ensures that a reasonable amount of the reference plane is visible between own-ship's horizontal position and the bottom of the screen.

Aircraft Symbol Scaling

Another adjustment to the perspective projection embodied in the frustum is the establishment of a normalized scaling for aircraft symbols. This scaling ensures that the screen size of the own-ship symbol is constant, and independent of the viewing geometry. The size of the own-ship symbol is defined as a user-selectable fraction of the screen width by specifying how many own-ship symbols placed end-to-end would reach from the left side to the right side of the screen. This is the same as specifying how many own-ship symbols placed end-to-end would reach from the left side to the right side of the frustum, on a line that is perpendicular to the viewing vector and which intersects the reference point (fig. 38). By scaling in this way, the own-ship symbol stays the same size on the screen, regardless of the distance between the eye point and own ship, or the size of the field-of-view angle. Thus, the space about own ship is altered by the various perspective parameters, but the displayed size of own ship is constant.

Other aircraft in the three-dimensional display space are scaled by the same amount as own ship but their projected screen sizes vary as a function of the viewing geometry. Aircraft symbols on the near-side of own ship range from nearly own-ship's size, for those located near own ship, to a size somewhat larger than own ship, as they approach the near clipping plane. Aircraft symbols on the far side of own-ship range from nearly own-ship's size, for those located close to ownship, to vanishingly small for those very close.
far away. The rate at which symbol sizes diminish with depth is a function of the perspective parameters.

Geometric Eyepoint versus User's Eye

When a perspective situation display is used as an aircraft instrument, there would be a severe limitation on the panel position of the display if the pilot's eye were required to be at the geometrically correct eyepoint in order to interpret the scene (fig. 39). Headup displays have this eyepoint limitation but only because they are used to superimpose graphics onto the visible scene outside the aircraft. A situation display provides a synthetic overview of the scene, not a window. Research is in progress (McGreevy and Ellis, 1984) to precisely determine the effects on spatial judgment of displacing the eye position from the geometric eyepoint.

PERSPECTIVE IN SPATIAL DISPLAYS

Spatial information displays can be implemented as alphanumeric lists of parameters, schematic diagrams, planviews or maps, perspective views, or even synthesized virtual environments. The displays that are easiest for designers to implement are, however, usually the most difficult for the users to interpret. For example, it is simple to display position information as a list of coordinates, but it is very difficult for a user to integrate and visualize the overall configuration of multiple objects, especially if the relative positions are changeable.

An alternative to encoding spatial information with lists of coordinates is to use an analog format. These range from very schematic formats to virtual replacements of the real visual environment. Schematic diagrams such as the Proximity Warning Indicator (PWI) (Senne, 1977) provide a low resolution, highly encoded representation that cannot clearly present information about multiple objects or trends and is, therefore, of little use for maintaining situation awareness. Maps and planviews clearly present two of the three dimensions of interest but encode the third one with shapes, numbers, or colors, and many of these planview maps also encode the object types. Perspective provides a pictorial analog of the space of interest where relative positions and velocities are presented in an integrated format that can be augmented with the use of metric symbology, optimized projection geometry, and more intuitive object symbols. Virtual environmental displays, which surround the user with an artificial visual world and most closely emulate natural perspective, may be useful for visual/audiopresence in activities such as proximity operations on orbit. Clearly, spatial information formats need not be limited to lists of coordinates.

Perspective is common to all pictorial spatial display techniques. To see something “in perspective” is to see it from a particular viewpoint. It is misleading to call any particular display technique “true 3-D,” because ultimately all visual information is projected to the eyes and is seen in perspective. The common feature of stereo, holographic, and varifocal mirror techniques is the presentation of multiple perspectives. Stereo consists of two perspective views seen simultaneously, one by each eye. Most of the benefit of stereo can be obtained by providing motion parallax, that is, by providing multiple perspectives over time.

A hologram effectively stores multiple perspectives which are sampled according to eye position. A single user can sample at most two of the perspectives at any given time, however, so all the other available perspectives and the computation that is required to generate them are wasted. The varifocal mirror technique involves sweeping out a virtual image, one two-dimensional slice at a time, which the eye integrates into a virtual three-dimensional image. This virtual object can be viewed from different positions, while providing different perspectives. The main advantage of this method is similar to that of holography: multiple perspectives are available simultaneously, and the same criticisms apply. Much of the same pictorial effect can be achieved using standard computer graphics displays by tracking the position of the viewer and correspondingly altering the perspective geometry.

Perspective spatial displays provide an integrated format for presenting three-dimensional information which is especially useful for presentation of dynamic spatial relations among several objects. The synthetic nature of computer-generated perspective spatial displays allows augmentation of viewpoint, symbology, scaling, and perspective geometry. The techniques described in this report apply equally well to all pictorial spatial displays.

The most important aspect of the design of a spatial information instrument is the quality of the information transfer (Billings and Cheaney, 1981) that it makes possible. The perspective geometry and symbology must be designed to optimize the communication of spatial information to the user. The flexible and interactive display described in this report has been a valuable tool for exploring designs for effective spatial information transfer.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, California 94035, January 4, 1985
REFERENCES

Billings, C. E.; and Cheaney, E. S., eds.: Information transfer problems in the aviation system. NASA TP-1875, 1981.


McGreevy, Michael W.: A perspective display of air traffic for the cockpit. MS report, University of California, Berkeley, May 1983.


Figure 1.— Perspective display.
Figure 2.— Planview display.
Figure 3.— Maneuver decisions.
Figure 4.— Aircraft seen in perspective.
Figure 5.— Scene with horizontal reference grid added.
Figure 6.— Scene with noselines showing grid position.
Figure 7.— Scene with metrics added to noselines.
Figure 8.— Scene with predictors added.
Figure 9.— Aircraft symbols for traffic display.
Parameters:

num: aircraft number (1..8);

xCoord: x coordinate of aircraft relative to ground-based origin (nautical miles);
positive x is East, negative x is West;

yCoord: y coordinate of aircraft relative to ground-based origin (nautical miles);
positive y is North, negative y is South;

altAgl: altitude of aircraft above the ground plane (feet);

hdg: heading angle of the aircraft relative to North;

ktSpd: speed of the aircraft in knots;

roll: roll angle of aircraft:
used to achieve a turn rate;
symbol rolled proportionately;

pitch: pitch angle of aircraft:
used to achieve a climb or descent rate;
symbol pitched proportionately;

type: aircraft symbol type:
could indicate (1) 727, (2) Cessna 150, (3) Jumbo jet, (4) UFO;

Format:

num  xCoord  yCoord  altAgl  hdg  ktSpd  roll  pitch  type
num  xCoord  yCoord  altAgl  hdg  ktSpd  roll  pitch  type

Example:

1  -4  -5  20000  60  400  5  0  1
2  -9  5  4000  -40  70  0  10  2
3  3  -1  6000  60  160  8  -5  3

(1 to 8 were needed per situation, but more are possible)

Figure 10.— Intruder aircraft situation definition file.
Figure 11.— Three different perspective views of the same traffic situation.
Parameters:

num: switch selection number (0..7);

dist: distance between ownship and the geometric eyepoint (0..32767);
Dist must be input as zero when range is non-zero.
Dist will be calculated from range and fov,
dist=range/[sin(fov/2)]

range: distance between ownship and the nearest clipping plane (0..32767);
Range must be input as zero when dist is non-zero.
Range will be calculated from dist and fov,
range=dist*sin(fov/2)

fov: field of view angle (1..179);

azi: horizontal angle of view vector relative to ownship heading (-180..180);

elev: vertical angle of view vector relative to ownship altitude plane (0..90);

Format:

num  dist  range  fov  azi  elev
-    -      -      -    -    -

Example:

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<th>num</th>
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<th>range</th>
<th>fov</th>
<th>azi</th>
<th>elev</th>
</tr>
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<td>6000</td>
<td>110</td>
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</tr>
</tbody>
</table>

Figure 12.— Display selection switch definition file.
PLANAR GEOMETRIC PROJECTIONS

PARALLEL

ORTHOGRAPHIC

TOP/FRONT/SIDE

ISOMETRIC

AXONOMETRIC

DIMETRIC

CAVALIER

TRIMETRIC

OBLIQUE

ONE POINT

TWO POINT

THREE POINT

PERSPECTIVE

Figure 13.— Planar geometric projections. (Carlborn and Paciorek, 1978)
(a) BOX OF VISIBLE SPACE (PARALLEL PROJECTIONS)

(b) FRUSTUM OF VISIBLE SPACE (CONVERGING PROJECTIONS)

Figure 14.—Visible space.
Figure 15.— Views along canonical axes of ownship.
Figure 16.—Types of perspective.
Figure 17.— Example of a display format that uses nonlinear projection.
Figure 18.— Light rays and the eye.

Figure 19.— Visible light rays.
Figure 20.— Plane segment P and the principle view vector.

Figure 21.— Bounding rays define the clipping planes.
Figure 22.— Plane segment $P$ and four clipping planes.

Figure 23.— Frustum of visible space.
Figure 24.— Eye-based (left-handed) coordinate system.

Figure 25.— Field-of-view angles.
Figure 26.— Perspective definition based on picture plane.
Figure 27.— Perspective definition based on reference point and clipping planes.
Figure 28. — Algebra of projection.

Figure 29. — Object is translated from definition origin to reference point.
Figure 30.— Object is rotated and scaled, and projected to the picture plane.
Figure 31.— Displayed view of the projected object.
Figure 32. — Example of clipping — side view.
**Abstract**

The design and implementation of a perspective display of air traffic for the cockpit is discussed. Parameters of the perspective are variable and interactive so that the appearance of the projected image can be widely varied. This approach makes allowances for exploration of perspective parameters and their interactions. The display was initially used to study the causes of horizontal maneuver biases found in experiments involving a plan view air traffic display format. Experiments to determine the effect of perspective geometry on spatial judgments have evolved from the display program.

The display depicts a volume of visible airspace around "own ship," the space around the aircraft symbol representing the pilot's own aircraft so that separation may be monitored in all directions. Symbols and metric aids which enhance the sense of space derived from perspective and which show both horizontal and vertical separation in an integrated format are contained in the display. By use of switches, the data tablet, and the computer terminal, the display may be altered in content, in direction of view, and in geometry of projection.

Perspective is one way that three-dimensional (3-D) information may be projected onto a two-dimensional (2-D) surface. Geometry of perspective can be described conveniently as a frustum whose position, orientation, and shape define the appearance of the projected image. The frustum shape embodies the bundle of light rays that produce a rectangular image. By defining the perspective algebraically as it is relative to the position of own ship, rather than as it is relative to the display "window," the visible space around own ship becomes part of the definition of the perspective. Several scaling techniques and other adjustments to the perspective are used to tailor the geometry for effective presentation of three-dimensional traffic situations.