

# Extending the Flicker Visibility Metric to a Range of Mean Luminance

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## Abstract

We previously proposed a flicker visibility metric for bright displays, based on psychophysical data collected at a high mean luminance. Here we extend the metric to other mean luminances. This extension relies on a linear relation between log sensitivity and critical fusion frequency, and a linear relation between critical fusion frequency and log retinal illuminance. Consistent with our previous metric, the extended flicker visibility metric is measured in just-noticeable differences (JNDs).

## Author Keywords

temporal; contrast sensitivity; flicker; light adaptation.

## 1. Objective and Background

Flicker is a perceptual attribute of displays that consists of an apparent fluctuation in brightness of a rapid periodic modulation of luminance. Display flicker is a significant potential defect of most current display technologies, because they periodically refresh the displayed image. Human visual sensitivity to variation of luminance over time is defined by the temporal contrast sensitivity function (TCSF). This function describes visual sensitivity as a function of temporal frequency of luminance modulation. The TCSF falls rapidly at high temporal frequencies, reaching a minimum of 1 at the Critical Fusion Frequency (CFF). Thus flicker is usually avoided if the display is refreshed at a rate above the CFF. However advanced display technologies such as scanning backlights, field emission displays, plasma displays, frame sequential color, and time-multiplexed stereo may introduce flicker artifacts (Zhang *et al.*, 2007).

We have previously proposed a metric for flicker visibility (Watson & Ahumada, 2011). That metric was based on a theoretical temporal contrast sensitivity function (TCSF) fit to psychophysical thresholds measured by (De Lange, 1958) for a 2 deg circular flickering disk on a uniform background of the same time average luminance. The theoretical function was based on a Gamma function impulse response (Watson, 1986).

The retinal illuminance used by De Lange in the data we used was 1000 Td, which is approximately that obtained in a 30 year old eye viewing a 100 deg<sup>2</sup> adapting field with a luminance of 60 cd m<sup>-2</sup> (Watson & Yellott, 2012). It is well known that the TCSF depends upon mean luminance (Watson, 1986). Indeed, de Lange himself produced data for a range of retinal illuminances. We have made use of those data to extend our metric to a range of mean luminance.

## 2. Methods

De Lange provided data for two observers at a range of adapting retinal illuminances (De Lange, 1958). We begin with the observation that at a given retinal illuminance, the high-frequency limb of the TCSF is approximately linear when log contrast sensitivity  $s$  is plotted against linear frequency  $f$ . If we transpose the data (exchange  $x$  and  $y$  coordinates), we have a graph of fusion frequency  $f$  plotted against log contrast sensitivity  $s$  (Figure 1). Note that log contrast sensitivity may also be regarded as -log contrast.

At each retinal illuminance, it is possible to fit these data with a straight line: a linear function of  $s$ . Since each straight line would have two parameters, this would result in a model with a number of parameters equal to 2 times the number of retinal illuminances.

To reduce the number of parameters, we have made use of the Ferry-Porter Law, which states that CFF is proportional to the log of the retinal illuminance (Tyler & Hamer, 1990). Thus at  $s = 0$  (a contrast of 1), the vertical intercept of each straight line should itself be a linear function of retinal illuminance. We assume in addition that the Ferry-Porter Law applies at lower contrasts. In particular, we assume that at  $s = 1$  (a contrast of 0.1) the intercepts are again a linear function of retinal illuminance. This allows us to use a linear model to also compute the slope and intercept of each line relating frequency to log sensitivity.

If we write  $s$  for log sensitivity and  $t$  for log retinal illuminance, and we write  $f(s,t)$  for the predicted fusion frequency (Hz) at the corresponding values of  $s$  and  $t$ , then we parameterize out bilinear model by the following four quantities:

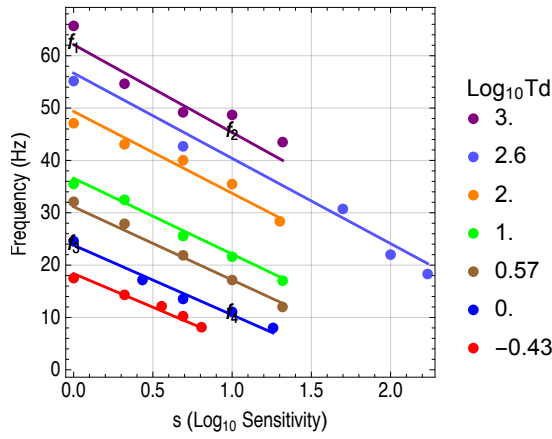
$$\begin{aligned} f_1 &= f(0,3) \\ f_2 &= f(1,3) \\ f_3 &= f(0,0) \\ f_4 &= f(1,0) \end{aligned} \quad (1)$$

These are four fusion frequencies, two for a contrast of 1 at a retinal illuminance of 1 Td, and the other two for a contrast of 0.1 at a retinal illuminance of 1000 Td. Then after algebraic manipulation it is possible to show that the bilinear formula for CFF is

$$\begin{aligned} f(s,t) &= f_3 + s(f_4 - f_3) \\ &+ 1/3 t(f_1 - f_3 + s(f_2 - f_1 + f_3 - f_4)) \end{aligned} \quad (2)$$

## 3. Results

We have fit this model to the high-frequency data of de Lange for observers L and V. We minimized the RMS error in the vertical (Hz) direction. We varied the number  $n$  of high frequencies, from the data set for each retinal illuminance, included in the fit. The results for observer L and  $n = 5$  are shown in Figure 1. The residual RMS error is 1.6 Hz. The estimated parameters are:  $f_1 = 62.1, f_2 = 45.3, f_3 = 23.8, f_4 = 10.5$ .



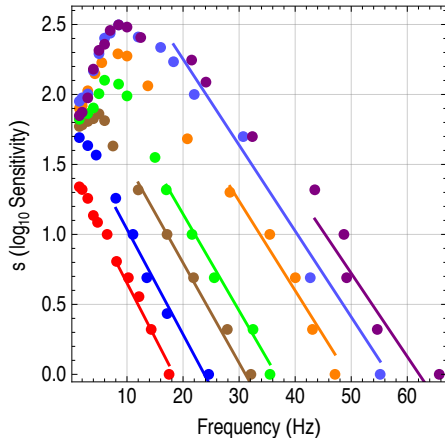
**Figure 1.** High frequency data of de Lange (1958) observer L and fit of the bilinear model for  $n = 5$ . The locations of the estimated parameters are also shown.

Equation 2 can be rearranged to compute the sensitivity as a function of frequency,

$$s(f, t) = \frac{3f + f_3(-3 + t) - f_1 t}{(f_3 - f_4)(-3 + t) + (-f_1 + f_2)t} \quad (3)$$

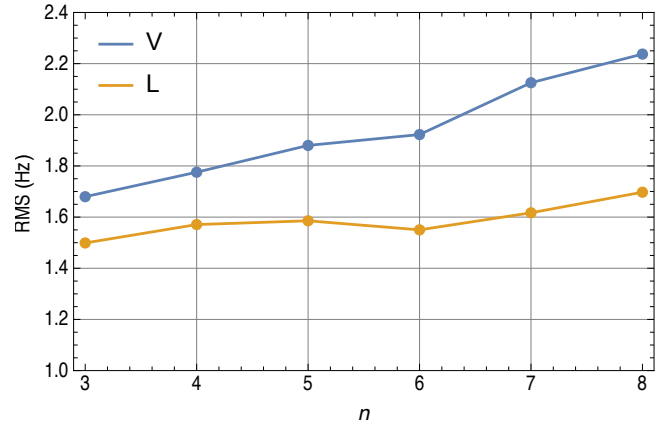
Since the flicker visibility metric is only concerned with the high-frequency portion of the TCSF, this formula suffices to compute TCSF values, and thereby to compute flicker visibility values in JNDs.

For clarity, we show in Figure 2 the more traditional view of temporal contrast sensitivity data along with predictions from Equation 3.



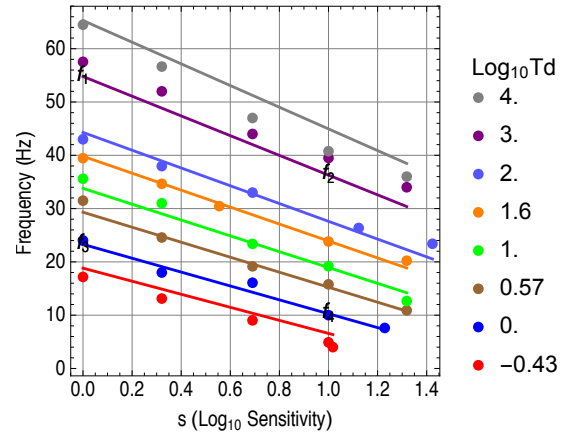
**Figure 2.** Data of de Lange (1958) observer L and the bilinear model sensitivity curves.

Looking at Figure 2 it is evident that the point at which the data depart from linearity is not clearly defined. In Figure 3 we show the RMS error of the fit for various values of  $n$ . The errors are higher, and climb more rapidly with  $n$ , for observer V. We select  $n = 5$  for further consideration, as a compromise between robust parameter estimation and minimizing error.



**Figure 3.** RMS error of fit of the bilinear model to data of de Lange (1958) observers V and L as a function of  $n$ , the number of high frequencies included in the fit.

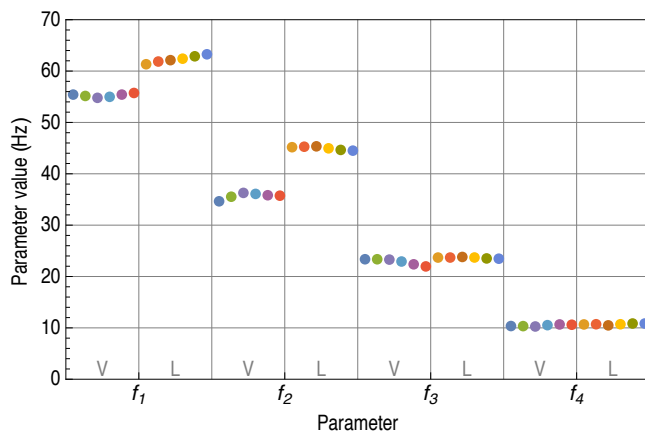
For comparison, we show the fit for observer V for  $n = 5$ , which suggests that much of the error is attributable to the highest retinal illuminance, which was not used with observer L.



**Figure 4.** Data of de Lange (1958) observer L and the bilinear model sensitivity curves.

In Figure 5 we show the dependence of parameter estimates on observer and on the value of  $n$ . The plot shows that the parameter estimates are quite robust and only weakly dependent on the value of  $n$ . The two observers do differ somewhat in their estimates for parameters  $f_1$  and especially  $f_2$ .

For reference, in Table 1 we provide parameter values for the two observers, based on  $n = 5$ .



**Figure 5.** Parameter estimates for observers L and V of de Lange (1958), as a function of  $n$ , the number of high frequencies included from the data set for each retinal illuminance. Each set of six successive points are for  $n=3, \dots, 8$ .

**Table 1.** Parameter estimates for the two observers of de Lange (1958), based on the highest five frequencies from each retinal illuminance.

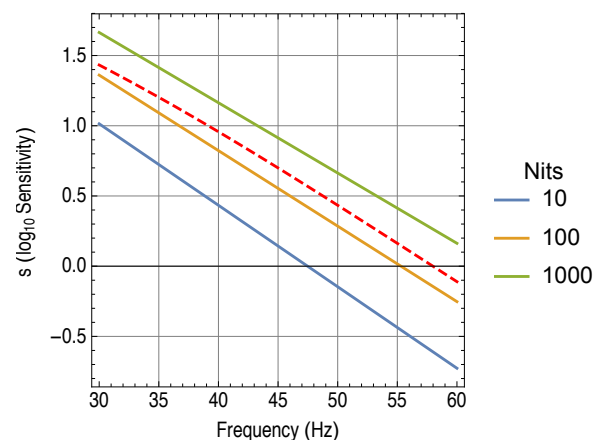
	Observer	
	V	L
$f_1$	54.8	62.1
$f_2$	36.3	45.3
$f_3$	23.3	23.8
$f_4$	10.3	10.5
RMS	1.88	1.58

#### 4. Discussion

We have provided a bilinear model of the high-frequency portion of the TCSF, based on data of de Lange, and on an assumption that the Ferry-Porter law remains valid for contrast values below 1. The model is a reasonable fit to the data.

In the above formulation, the quantity  $t$  refers to the log of the retinal illuminance. Typically, retinal illuminance is not known, but rather the display luminance is specified. Using a recent formula, we can compute an expected retinal illuminance from the display luminance and other contextual information (Watson & Yellott, 2012).

In the following example, we assume that a display of 314 square degrees is viewed binocularly in an otherwise darkened room by an observer of age 30. Under these conditions, from the luminance we can compute the predicted pupil diameter, and thence the retinal illuminance. From that we can predict the expected sensitivity at high frequencies, as shown in Figure 6. For comparison, we also show the predictions from the earlier model of Watson and Ahumada (2011) designed for moderate to high luminances.



**Figure 6.** Predicted sensitivity at high frequencies for observer V of de Lange (1958) for three display luminances. The dashed red curve is the prediction from (Watson & Ahumada, 2011).

Farrell and colleagues (1986; 1987), using a somewhat different approach, also proposed a flicker metric for arbitrary display luminances, but their metric did not exploit the Ferry-Porter Law. In future work we will compare their metric to ours.

To advocate a particular version of this metric as a standard, it will be necessary to resolve the differing parameters for the two observers, or perhaps to collect new data on a larger set of observers.

#### 5. Acknowledgements

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#### 6. References

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