VISUAL OPTIMIZATION OF DCT QUANTIZATION MATRICES FOR INDIVIDUAL IMAGES

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ABSTRACT

Many image compression standards (JPEG, MPEG, H.261) are based on the Discrete Cosine Transform (DCT). However, these standards do not specify the actual DCT quantization matrix. We have previously provided mathematical formulae to compute a perceptually lossless quantization matrix. Here I show how to compute a matrix that is optimized for a particular image. The method treats each DCT coefficient as an approximation to the local response of a visual "channel." For a given quantization matrix, the DCT quantization errors are adjusted by contrast sensitivity, light adaptation, and contrast masking, and are pooled non-linearly over the blocks of the image. This yields an 8x8 "perceptual error matrix." A second non-linear pooling over the perceptual error matrix yields total perceptual error. With this model we may estimate the quantization matrix for a particular image that yields minimum bit rate for a given total perceptual error, or minimum perceptual error for a given bit rate. Custom matrices for a number of images show clear improvement over image-independent matrices. Custom matrices are compatible with the JPEG standard, which requires transmission of the quantization matrix.

2. IMAGE-INDEPENDENT PERCEPTUAL QUANTIZATION

The JPEG QM is not defined by the standard, but is supplied by the user and stored or transmitted with the compressed image. The principle that should guide the design of a JPEG QM is that it provide optimum visual quality for a given bit rate. QM design thus depends upon the visibility of quantization errors at the various DCT frequencies. In recent papers, Peterson et al. 3, 4 have provided measurements of threshold amplitudes for DCT basis functions. For each frequency ij they measured psychophysically the smallest coefficient that yielded a visible signal. Call this threshold $t_{ij}$. From Eqns (1) and (2) it is clear that the maximum possible quantization error $e_{ijk} = q_{ij} / 2$. Thus to ensure that all errors are invisible (below threshold), we set

$$q_{ij} = 2t_{ij}.$$  

I call this the Image-Independent Perceptual approach (IIP). It is perceptual because it depends explicitly upon detection thresholds for DCT basis functions, but is image-independent because a single matrix is computed independent of any image. Ahumada et al. 5, 6 have extended the value of this approach by measuring $t_{ij}$ under various conditions and by providing a formula that allows extrapolation to other display luminances ($L$) and pixel sizes ($px,py$), as well as other display properties. For future reference, we write this formula in symbolic form as

$$t_{ij} = apw[i,j,L,px,py,...].$$  

3. LIMITATIONS OF THE IMAGE-INDEPENDENT APPROACH

While a great advance over the ad hoc matrices that preceded it, the IIP approach has several shortcomings. The fundamental drawback is that the matrix is computed independent of the image. This would not be a problem if visual thresholds for artifacts were fixed and independent of the image upon which they are superimposed, but this is not the case.

First, visual thresholds increase with background luminance., and variations in local mean luminance within the image will in fact produce
substantial variations in DCT threshold. We call this 
luminance masking.

Second, threshold for a visual pattern is typically
reduced in the presence of other patterns, particularly
those of similar spatial frequency and orientation, a
phenomenon usually called contrast masking. This
means that threshold error in a particular DCT
coefficient in a particular block of the image will be a
function of the value of that coefficient in the original
image.

Third, the IIP approach ensures that any single
error is below threshold. But in a typical image there
are many errors, of varying magnitudes. The
visibility of this error ensemble is not generally equal
| TO THE VISIBILITY OF THE LARGEST ERROR, BUT REFLECTS A
| POOLING OF ERRORS, OVER BOTH FREQUENCIES AND BLOCKS OF
| THE IMAGE. I CALL THIS ERROR POOLING.

Fourth, when all errors are kept below a
perceptual threshold a certain bit rate will result. The
IIP method gives no guidance on what to do when a
lower bit rate is desired. The ad hoc "quality factors"
employed in some JPEG implementations, which
usually do no more than multiply the quantization
matrix by a scalar, will allow an arbitrary bit rate, but
do not guarantee (or even suggest) optimum quality
at that bit rate. I call this the problem of selectable
quality.

4. IMAGE-DEPENDENT APPROACH

Here I present a general method of designing a
custom quantization matrix tailored to a particular
image. This image-dependent perceptual (IDP) method
incorporates solutions to each of the problems
described above. The strategy is to develop a very
simple model of perceptual error, based upon DCT
coefficients, and to iteratively estimate the
quantization matrix which yields a designated
perceptual error or bit-rate. We call this the DCTune
algorithm, because it tunes the DCT quantization
matrix to the individual image.

Luminance Masking

Detection threshold for a luminance pattern
typically depends upon the mean luminance of the
local image region: the brighter the background, the
higher the luminance threshold. This is usually
called "light adaptation," but here we call it
"luminance masking" to emphasize the similarity to
contrast masking, discussed in the next section.

Figure 1. Log of $t_{ij}$ as a function of luminance $L$ of
the block. From the top, the curves are for
frequencies of \{7,7\}, \{0,7\}, \{0,0\}, \{0,3\}, and \{0,1\}.
The dashed curves are the power function
approximation described in the text.

To illustrate this effect, the solid lines in Fig. 1 plot values of the formula for $t_{ij}$ provided by
Ahumada and Peterson as a function of the mean
display luminance of the block, assuming that the maximum
display luminance is 100 cd m$^{-2}$ and that the
greyscale resolution is 8 bits. The three curves are for
five representative frequencies. These curves
illustrate that variations by as much as 0.5 log unit in
$t_{ij}$ might be expected to occur within an image, due to
variations in the mean luminance of the block.

We can compute a luminance-masked threshold
matrix for each block in either of two ways. The first
is to make use of a formula such as that supplied by
Ahumada, Peterson, and Watson:

\[
t_{ijk} = a_p w[i, j, L_0, c_{00k}/c_{00}] \times (L_0)^c_{00}
\]

where $c_{00k}$ is the DC coefficient of the DCT for
block $k$, $L_0$ is the mean luminance of the display, and
$c_{00}$ is the DC coefficient corresponding to $L_0$ (1024 for
an 8 bit image).

A second, simpler solution is to approximate the
dependence of $t_{ij}$ upon $c_{00k}$ with a power function:

\[
t_{ijk} = t_{ijc}(c_{00k}/c_{00})^{a_T}
\]

This approximation is illustrated by the dashed
lines in Fig. 1. The initial calculation of $t_{ij}$ should
be made assuming a display luminance of $L_0$. The
parameter $a_T$ takes its name from the corresponding
parameter in the formula of Ahumada and Peterson,
wherein they suggest a value of 0.649. Note that luminance masking may be suppressed by setting $a_x = 0$. More generally, $a_x$ controls the degree to which this masking occurs. Note also that the power function makes it easy to incorporate a non-unit display Gamma, by multiplying $a_x$ by the Gamma exponent.

**Contrast Masking**

Contrast masking refers to the reduction in the visibility of one image component by the presence of another. Here we consider only masking within a block and a particular DCT coefficient. We employ a model of visual masking that has been widely used in vision models,\(^11,12\). Given a DCT coefficient $c_{ijk}$ and a corresponding absolute threshold $t_{ijk}$ our masking rule states that the masked threshold $m_{ijk}$ will be

$$m_{ijk} = \text{Max}\left[t_{ijk}, c_{ijk} w_{ij} t_{ijk}^{1-w_{ij}}\right]$$

where $w_{ij}$ is an exponent that lies between 0 and 1. Because the exponent may differ for each frequency, we allow a matrix of exponents equal in size to the DCT. Note that when $w_{ij} = 0$, no masking occurs, and the threshold is constant at $t_{ijk}$. When $w_{ij} = 1$, we have what is usually called "Weber Law" behavior, and threshold is constant in log or percentage terms (for $c_{ijk} > t_{ijk}$). The function is pictured for a typical empirical value of $w_{ij} = 0.7$ in Fig. 2.

![Figure 2. Contrast masking function, describing the masked threshold $m_{ijk}$ as a function of DCT coefficient $c_{ijk}$, for parameters $w_{ij} = 0.7$, $t_{ijk} = 2$.](image)

Because the effect of the DC coefficient upon thresholds has already been expressed by luminance masking, we specifically exclude the DC term from the contrast masking, by setting the value of $w_{00} = 0$.

**Perceptual Error and Just-Noticeable-Differences**

In vision science, we often express the magnitude of a signal in multiples of the threshold for that signal. These threshold units are often called "just-noticeable differences," or jnd’s. Having computed a masked threshold $m_{ijk}$, the error DCT may therefore be expressed in jnd’s as

$$d_{ijk} = e_{ijk} / m_{ijk}$$

(8)

**Spatial Error Pooling**

To pool the errors in the jnd DCT we employ another standard feature of current vision models: the so-called Minkowski metric. It often arises from an attempt to combine the separate probabilities that individual errors will be seen, in the scheme known as "probability summation"\(^13\). We pool the jnds for a particular frequency $[i,j]$ over all blocks $k$ as

$$p_{ij} = \left(\sum_k |d_{ijk}|^\beta_s\right)^{1/\beta_s}$$

(9)

In psychophysical experiments that examine summation over space a $\beta_s$ of about 4 has been observed\(^13\). The exponent $\beta_s$ is given here as a scalar, but may be made a matrix equal in size to the QM to allow differing pooling behavior for different DCT frequencies. This matrix $p_{ij}$ is now a simple measure of the visibility of artifacts within each of the frequency bands defined by the DCT basis functions. I call it the "perceptual error matrix."

**Frequency Error Pooling**

This perceptual error matrix $p_{ij}$ may itself be of value in revealing the frequencies that result in the greatest pooled error for a particular image and quantization matrix. But to optimize the matrix we would like a single-valued perceptual error metric. We obtain this by combining the elements in the perceptual error matrix, using a Minkowski metric with a possibly different exponent, $\beta_f$

$$P = \left(\sum_{ij} p_{ij}^{\beta_f}\right)^{1/\beta_f}$$

(10)

It is now straightforward, at least conceptually, to optimize the quantization matrix to obtain minimum bit-rate for a given $P$, or minimum $P$ for a given bit rate. In practice, however, a solution may be difficult to compute. But if $\beta_f = \infty$, then $P$ is given by the maximum of the $p_{ij}$. Under this condition minimum bit-rate for a given $P=\psi$ is achieved when all $p_{ij}=\psi$.

**Optimization Method**

Under the assumption $\beta_f = \infty$, the joint optimization of the quantization matrix reduces to the vastly simpler separate optimization of the
individual elements of the matrix. Each entry of the
perceptual error matrix $p_{ij}$ may be considered an
independent monotonically increasing function of
the corresponding entry $q_{ij}$ of the quantization
matrix.

In the following examples, unless otherwise
stated, the parameter values used were $a_T = 0.649$, $\beta = 4$, $w_{ij} = 0.7$, display mean luminance $L_0 = 65 \text{ cd m}^{-2}$, image greylevels = 256, $c_{00} = 1024$. The viewing
distance was assumed to yield 32 pixels/degree. For
a 256 by 256 pixel image, this corresponds to a
viewing distance of 7.115 picture heights.

**Optimizing QM for a given bit-rate**

It is of interest to relate the JPEG bit-rate to the
perceptual error level $\psi$. This is shown for the Lena
and Mandrill images in Fig. 3. This is a sort of inverse
"rate-distortion" function. Note that useful bit-rates
below 2 bits/pixel yield perceptual errors above
about 2.

To obtain a QM that yields a given bit rate $h_0$
with minimum perceptual error $\psi$ we note that the
bit rate is a decreasing function of $\psi$, as shown in
Fig. 3.

![Figure 3. JPEG bit-rate versus perceptual error $\psi$ for
the Lena (lower curve) and Mandrill (upper curve) images.]

The most meaningful contest between IDP and
IIP approaches is to compare images compressed by
the two methods to a constant low bit rate. Figure 4
shows that the IDP method is superior, even in
relatively low-quality printed renditions.

![Figure 4. IIP (top) and IDP (bottom) compressions at
0.25 bits/pixel.]

**5. APPLICATION TO SPACE IMAGERY**

Image compression will play a vital role in the
distribution of preview images of science data to
scientists at distributed sites, especially in programs
such as EOS and the Mission to Planet Earth\footnote{14}. Due
to the generally high performance and wide
availability of the JPEG still image compression
standard, we expect it to play an important role in
this area. Since the JPEG standard includes the
quantization matrix as part of the file, DCTune
technology provides a method of optimizing the bit-
rate/quality trade-off for each science image.
Lossy image compression based on the DCT may also play a role in the recovery of scientific imagery from spacecraft. The Galileo orbiter spacecraft is now on its way to Jupiter. Due to a malfunction of the main antenna, image data will be sent to earth over an auxiliary antenna with approximately 15,000 times lower bandwidth. Image compression will be used to partially compensate for the loss of bandwidth. In support of this effort, we have designed quantization matrices for use in the Galileo mission, based on application of DCTune technology to existing Voyager and Galileo images.

An example of DCTune algorithm applied to an image of Jupiter obtained by the previous Voyager mission is shown in Fig. 5. It shows the original and three levels of optimized compression: 1.0, 0.5, and 0.25 bits/pixel.

6. SUMMARY

I have shown how to compute a visually optimal quantization matrix for a given image. These image-dependent quantization matrices produce better results than image independent matrices. The DCTune algorithm can be easily incorporated into JPEG-compliant applications.
In a practical sense, the DCTune method proposed here solves two problems. The first is to provide maximum visual quality for a given bit rate. The second problem it solves is to provide the user with a sensible and meaningful quality scale for JPEG (or other DCT-based) compression. Without such a scale, each image must be repeatedly compressed, reconstructed, and evaluated by eye to find the desired level of visual quality.

7. ACKNOWLEDGMENTS

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8. REFERENCES


