DCTune: A TECHNIQUE FOR VISUAL OPTIMIZATION OF DCT QUANTIZATION MATRICES FOR INDIVIDUAL IMAGES.

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1. ABSTRACT

Image compression standards based on the Discrete Cosine Transform do not specify the quantization matrix. This matrix should be designed to provide maximum visual quality for minimum bitrate. I show how this goal can be achieved for individual images in the context of a DCT-based model of visual quality.

2. JPEG DCT QUANTIZATION

The JPEG image compression standard provides a mechanism by which images may be compressed and shared among users. The image is first divided into blocks of size 8 x 8. Each block is transformed into its DCT, which we write as $c_{ijk}$, where $i,j$ indexes the DCT frequency (or basis function), and $k$ indexes a block of the image. Each block is then quantized by dividing it, coefficient by coefficient, by a quantization matrix (QM) $q_{ij}$, and rounding to the nearest integer

$$u_{ijk} = \text{Round}\left[ \frac{c_{ijk}}{q_{ij}} \right].$$

The quantization error $e_{ijk}$ in the DCT domain is then

$$e_{ijk} = c_{ijk} - u_{ijk}q_{ij}. $$

3. IMAGE-INDEPENDENT PERCEPTUAL QUANTIZATION

The JPEG QM is not defined by the standard, but is supplied by the user and stored or transmitted with the compressed image. The principle that should guide the design of a JPEG QM is that it provide optimum visual quality for a given bit rate. QM design thus depends upon the visibility of quantization errors at the various DCT frequencies. In recent papers, Peterson et al. have provided measurements of threshold amplitudes for DCT basis functions. For each frequency $i$, if they measured psychophysically the smallest coefficients that yielded a visible signal, call this threshold $t_{ij}$. From Eqs. (1) and (2) it is clear that the maximum possible quantization error $e_{ijk}$ is $q_{ij}/2$. Thus to ensure that all errors are invisible (below threshold), we set

$$q_{ij} = 2t_{ij}. $$

I call this the Image-Independent Perceptual approach (IIP). It is perceptual because it depends explicitly upon detection thresholds for DCT basis functions, but is image-independent because a single matrix is computed independent of any image. Ahumada et al. have extended the value of this approach by measuring $t_{ij}$ under various conditions and by providing a formula that allows extrapolation to other display luminances ($L$) and pixel sizes ($px,py$), as well as other display properties. For future reference, we write this formula in symbolic form as

$$t_{ij} = \text{ap}[i, j, L, px, py, ...]$$

4. LIMITATIONS OF THE IIP APPROACH

While a great advance over the ad hoc matrices that preceded it, the IIP approach has several shortcomings. The fundamental drawback is that the matrix is computed independent of the image. This would not be a problem if visual thresholds for artifacts were fixed and independent of the image upon which they are superimposed, but this is not the case.

First, visual thresholds increase with background luminance, and variations in local mean luminance within the image will in fact produce substantial variations in DCT threshold. We call this luminance masking.

Second, threshold for a visual pattern is typically reduced in the presence of other patterns, particularly those of similar spatial frequency and orientation, a phenomenon usually called contrast masking. This means that threshold error in a particular DCT coefficient in a particular block of the image will be a function of the value of that coefficient in the original image.

Third, the IIP approach ensures that any single error is below threshold. But in a typical image there are many errors, of varying magnitudes. The visibility of this error ensemble is not generally equal to the visibility of the largest error, but reflects a pooling of errors, over both frequencies and blocks of the image. I call this error pooling.

Fourth, when all errors are kept below a perceptual threshold a certain bit rate will result. The IIP method gives no guidance on what to do when a lower bit rate is desired. The ad hoc “quality factors” employed in some JPEG implementations, which usually do no more than multiply the quantization matrix by a scalar, will allow an arbitrary bit rate, but do not guarantee (or even suggest) optimum quality at that bit rate. I call this the problem of selectable quality.

Here I present a general method of designing a custom quantization matrix tailored to a particular image. This image-dependent perceptual (IDP) method incorporates solutions to each of the problems described above. The strategy is to develop a very simple model of perceptual
error, based upon DCT coefficients, and to iteratively estimate the quantization matrix which yields a designated perceptual error.

5. LUMINANCE MASKING

Detection threshold for a luminance pattern typically depends upon the mean luminance of the local image region: the brighter the background, the higher the luminance threshold. This is usually called “light adaptation,” but here we call it "luminance masking" to emphasize the similarity to contrast masking, discussed in the next section.

\[ t_{ijk} = a_p[i,j,L_0 c_{00k}/c_{00}] \]  

(5)

where \( c_{00k} \) is the DC coefficient of the DCT for block \( k \), \( L_0 \) is the mean luminance of the display, and \( c_{00} \) is the DC coefficient corresponding to \( L_0 \) (1024 for an 8 bit image).

A second, simpler solution is to approximate the dependence of \( t_{ij} \) upon \( c_{00k} \) with a power function:

\[ t_{ijk} = t_{ij}(c_{00k}/\bar{c}_{00})^{a_T} \]  

(6)

This approximation is illustrated by the dashed lines in Fig. 1. The initial calculation of \( t_{ij} \) should be made assuming a display luminance of \( L_0 \). The parameter \( a_T \) takes its name from the corresponding parameter in the formula of Ahumada and Peterson, wherein they suggest a value of 0.649. Note that luminance masking may be suppressed by setting \( a_T = 0 \). More generally, \( a_T \) controls the degree to which this masking occurs. Note also that the power function makes it easy to incorporate a non-unity display Gamma, by multiplying \( a_T \) by the Gamma exponent.

6. CONTRAST MASKING

Contrast masking refers to the reduction in the visibility of one image component by the presence of another. Here we consider only masking within a block and a particular DCT coefficient. We employ a model of visual masking that has been widely used in vision models.

\[ m_{ijk} = \text{Max}\left[t_{ijk}, c_{ijk}^{W_{ij}} t_{ijk}^{1-W_{ij}}\right] \]  

(7)

where \( W_{ij} \) is an exponent that lies between 0 and 1. Because the exponent may differ for each frequency, we allow a matrix of exponents equal in size to the DCT. Note that when \( W_{ij} = 0 \), no masking occurs, and the threshold is constant in log or percentage terms (for \( c_{ijk} > t_{ijk} \)). The function is pictured for a typical empirical value of \( W_{ij} = 0.7 \) in Fig. 2.

A contrast masking function, describing the masked threshold \( m_{ijk} \) as a function of DCT coefficient \( c_{ijk} \), for parameters \( W_{ij} = 0.7, t_{ijk} = 2 \).

Because the effect of the DC coefficient upon thresholds has already been expressed by luminance masking, we specifically exclude the DC term from the contrast masking, by setting the value of \( W_0 = 0 \).
7. PERCEPTUAL ERROR AND JUST-NOTICEABLE-DIFFERENCES

In vision science, we often express the magnitude of a signal in multiples of the threshold for that signal. These threshold units are often called “just-noticeable differences,” or jnd’s. Having computed a masked threshold $m_{ijk}$, the error DCT may therefore be expressed in jnd’s as

$$d_{ijk} = e_{ijk} / m_{ijk}$$

(8)

8. SPATIAL ERROR POOLING

To pool the errors in the jnd DCT we employ another standard feature of current vision models: the so-called Minkowski metric. It often arises from an attempt to combine the separate probabilities that individual errors will be seen, in the scheme known as “probability summation” $^{11}$. We pool the jnds for a particular frequency $(i,j)$ over all blocks $k$ as

$$p_{ij} = \left( \sum_k d_{ijk} \right)^{\beta_s}$$

(9)

In psychophysical experiments that examine summation over space a $\beta_s$ of about 4 has been observed $^{11}$. The exponent $\beta_s$ is given here as a scalar, but may be made a matrix equal in size to the QM to allow differing pooling behavior for different DCT frequencies. This matrix $p_{ij}$ is now a simple measure of the visibility of artifacts within each of the frequency bands defined by the DCT basis functions. I call it the “perceptual error matrix.”

9. FREQUENCY ERROR POOLING

This perceptual error matrix $p_{ij}$ may itself be of value in revealing the frequencies that result in the greatest pooled error for a particular image and quantization matrix. But to optimize the matrix we would like a single-valued perceptual error metric. We obtain this by combining the elements in the perceptual error matrix, using a Minkowski metric with a possibly different exponent, $\beta_f$

$$P = \left( \sum_{ij} p_{ij}^{\beta_f} \right)^{1/\beta_f}$$

(10)

It is now straightforwad, at least conceptually, to optimize the quantization matrix to obtain minimum bit-rate for a given $P$, or minimum $P$ for a given bit rate. In practice, however, a solution may be difficult to compute. But if $\beta_f = \infty$, then $P$ is given by the maximum of the $p_{ij}$. Under this condition minimum bit-rate for a given $P = \psi$ is achieved when all $p_{ij} = \psi$.

10. OPTIMIZATION METHOD

Under the assumption $\beta_f = \infty$, the joint optimization of the quantization matrix reduces to the vastly simpler separate optimization of the individual elements of the matrix. Each entry of the perceptual error matrix $p_{ij}$ may be considered an independent monotonically increasing function of the corresponding entry $q_{ij}$ of the quantization matrix.

In the following examples, unless otherwise stated, the parameter values used were $a_T = 0.649$, $\beta = 4$, $w_{ij} = 0.7$, display mean luminance $L_0 = 65 \text{ cd m}^{-2}$, image greylevels = 256, $c_{100} = 1024$. The viewing distance was assumed to yeild 32 pixels/degree. For a 256 by 256 pixel image, this corresponds to a viewing distance of 7.115 picture heights.

11. OPTIMIZING QM FOR A GIVEN BIT-RATE

It is of interest to relate the JPEG bit-rate to the perceptual error level $\psi$. This is shown for the Lena and Mandrill images in Fig. 3. This is a sort of inverse “rate-distortion” function. Note that useful bit-rates below 2 bits/pixel yield perceptual errors above about 2.

Figure 3. JPEG bit-rate versus perceptual error $\psi$ for the Lena (lower curve) and Mandrill (upper curve) images.

To obtain a QM that yields a given bit rate $h_0$ with minimum perceptual error $\psi$ we note that the bit rate is a decreasing function of $\psi$, as shown in Fig. 3.

The most meaningful contest between IDP and IIP approaches is to compare images compressed by the two methods to a constant low bit rate. Figure 4 shows that the IDP method is superior, even in relatively low-quality printed renditions.
13. ACKNOWLEDGMENTS

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14. REFERENCES


