

A formula for the mean human optical modulation transfer function as a function of pupil size

Andrew B. Watson

NASA Ames Research Center, Moffett Field, CA, USA



We have constructed an analytic formula for the mean radial modulation transfer function of the best-corrected human eye as a function of pupil diameter, based on previously collected wave front aberrations from 200 eyes (Thibos, Hong, Bradley, & Cheng, 2002). This formula will be useful in modeling the early stages of human vision.

Introduction

Modeling of human vision begins with an optical component. In the past, optical effects have frequently been ignored or regarded as part of the contrast sensitivity function. In recent years there has been a growing appreciation of the specific role of optics in spatial vision and an increasing need for a simple model for visual optics. In particular, there is a need for a model of the human visual point spread function (PSF), or its Fourier transform, the optical transfer function (OTF), either of which would allow calculation of retinal images of visual displays. Because the PSF depends significantly upon the pupil diameter, the model should include this parameter.

While actual individual optical PSFs are not symmetrical, their average is unlikely to exhibit any phase shifts, in which case the average OTF would be all real and equivalent to its modulation transfer function (MTF). Likewise, while individual MTFs are not radially symmetric, their average is likely to be approximately symmetric. In this report we develop a mathematical formula for the average human visual radial MTF as a function of pupil diameter.

Estimates of the OTF or MTF of the human eye have employed double-pass (Campbell & Gubisch, 1966; Westheimer & Campbell, 1962), interferometric (Campbell & Green, 1965; Williams, Brainard, McMahon, & Navarro, 1994), and aberrometric methods (Thibos, Hong, Bradley, & Cheng, 2002; Walsh & Charman, 1988). Here we use the last method,

making use of aberration data collected elsewhere (Thibos et al., 2002).

A number of analytical formulas have been proposed for the MTF or PSF (Artal & Navarro, 1994; Deeley, Drasdo, & Charman, 1991; Geisler, 1984; Guirao et al., 1999; IJspeert, van den Berg, & Spekreijse, 1993; Jennings & Charman, 1997; Krueger & Moser, 1973; Navarro, Artal, & Williams, 1993; Williams, Brainard, McMahon, & Navarro, 1994). Of these, only two provide formulas for various pupils and white light (Deeley et al., 1991; IJspeert et al., 1993). The former is based largely on double-pass data from three observers. The latter formula is particularly elaborate, including effects of age and pigmentation but is based (for small angles) on the data of one observer. Our formula is an advance upon existing formulas for various pupils in white light in being based on modern measurements from a large population of eyes.

Our approach is enabled by three recent developments. The first is the collection of wave front aberration data, for one wavelength and pupil diameter, from 200 best-corrected eyes of 100 young visually healthy observers (Thibos et al., 2002). The second development consist of mathematical techniques for computing aberrations at a smaller pupil from those collected at a larger pupil (Díaz, Fernández-Dorado, Pizarro, & Arasa, 2009; Mahajan, 2010; Schwiegerling, 2002). This allows us to extend the data of Thibos et al. (2002) to a range of pupil sizes. The third development consists of a mathematical technique for computing the polychromatic PSF from monochromatic aberrations (Ravikumar, Thibos, & Bradley, 2008). Together these three developments allow us to compute the white light MTF for a large population of individual eyes at various pupil diameters and from their average to derive an analytical formula. In conjunction with a recently proposed formula for average pupil diameter (Watson & Yellott, 2012), we may now compute the average MTF for specified viewing conditions.

Citation: Watson, A. (2013). A formula for the mean human optical modulation transfer function as a function of pupil size. *Journal of Vision*, 13(6):18, 1–11, <http://www.journalofvision.org/content/13/6/18>, doi:10.1167/13.6.18.

Computing the mean radial MTF

The two-dimensional complex OTF can be computed from a particular set of wave front aberrations obtained for a given wavelength and pupil diameter. Here we employ established techniques to extend this calculation to other pupil diameters and to white light. We apply these techniques to a set of 200 eyes for which aberration data were collected by Thibos et al. (2002). We compute the MTF, its radial average, and then the population radial average. We then approximate the population radial average with a mathematical formula.

The data we used in this study consisted of wave front aberrations for 200 eyes from 100 individuals with normal healthy vision (Thibos et al., 2002). The average age of the subjects was 26.1 years. Wave fronts were measured at a wavelength of 633 nm and referenced to a 6 mm pupil. Subjectively measured sphero-cylindrical refractive errors were corrected before the wave front measurement, but subsequent measurements included residual lower order aberrations that we included in our calculations. Each set of aberrations was expressed as Zernike coefficients for the first 35 modes.

To compute the mean radial MTF for each pupil diameter, we performed the following steps.

1. For each of the 200 eyes, starting from the Zernike coefficients at 6 mm, and using the methods described by Schwiegerling (2002) (see also Díaz et al., 2009; Mahajan, 2010), coefficients were computed for pupil diameters from 2 to 6 mm in steps of 0.5 mm.
2. For each eye and pupil diameter we then computed from the Zernike coefficients the discrete OTF for polychromatic (equal energy spectrum) white light, using the methods described in Watson and Ahumada (2008, 2012) and based on methods of Ravikumar et al. (2008). Zernike radial orders of two or greater were used. We used a center wavelength of 555 nm and a wavelength spacing of 20 nm, weighted by luminance. The discrete OTF was 256×256 in size. The scale of the discrete OTF and PSF depends upon pupil diameter. Pupil diameters from 2 to 6 mm yield PSF image resolutions of 521 to 174 samples/deg and OTF resolutions of 2.035 to 0.678 samples/cycle/deg, respectively.
3. For each OTF, we then computed the MTF as the absolute value of the OTF.
4. For each MTF, we then computed the radial MTF. For each pixel in the MTF we computed its distance from the origin in cycles/deg. The complete set of pixels was then binned by distance with a bin width of 1 cycle/deg. The magnitudes within each bin were averaged. The result was a vector of magnitudes and associated spatial frequencies.

5. Finally, we computed the mean radial MTF, computing the mean and standard deviation at each frequency over the 200 eyes. We considered averages of both log and linear values, but because the distributions of log values were much more skewed than linear values, we used the linear mean.

Our method of computing the radial mean from a set of OTFs (Steps 3–5) is essentially “method D” discussed by Thibos et al. (2002). The results are shown separately for each pupil diameter in Figure 1. To allow a more direct comparison of the various pupil diameters, we also plot the means together in Figure 2.

Fitting formulas to the mean radial MTF

Previously, for each pupil diameter we had computed the mean and standard deviation (over eyes) of the magnitudes at each radial spatial frequency. Candidate functions were fit to the mean radial MTF for each pupil diameter by minimizing the sum over all radial frequencies of squared errors between the function and the mean at each frequency, divided by the corresponding variance at each frequency. At each pupil diameter, fitting was restricted to magnitudes greater than 0.01. This means that there may be different numbers of magnitudes at different pupil diameters. One or more parameters were estimated for each function for each pupil diameter. Finally, we computed an *RMS* error for the complete set of pupil diameters by summing the minimum squared errors, dividing by the total number of data points, and taking the square root.

A number of formulas for the human average MTF have been proposed. Jennings and Charman (1997) provide a table of examples, which we extend here. These functions may be divided into two groups, which we describe as the exponential group and the Lorentzian group. Some of the formulas also make use of the diffraction-limited MTF, which we discuss first.

Diffraction limited MTF

The MTF of an aberration-free incoherent optical system limited only by diffraction, at a wavelength λ nm and with a circular pupil with diameter d mm is given by

$$D(u, d, \lambda) = \begin{cases} \frac{2}{\pi} \left(\cos^{-1}(\hat{u}) - \hat{u} \sqrt{1 - \hat{u}^2} \right) & \hat{u} < 1 \\ 0 & \hat{u} \geq 1 \end{cases}, \quad (1)$$

where u is spatial frequency in cycles/deg, and where

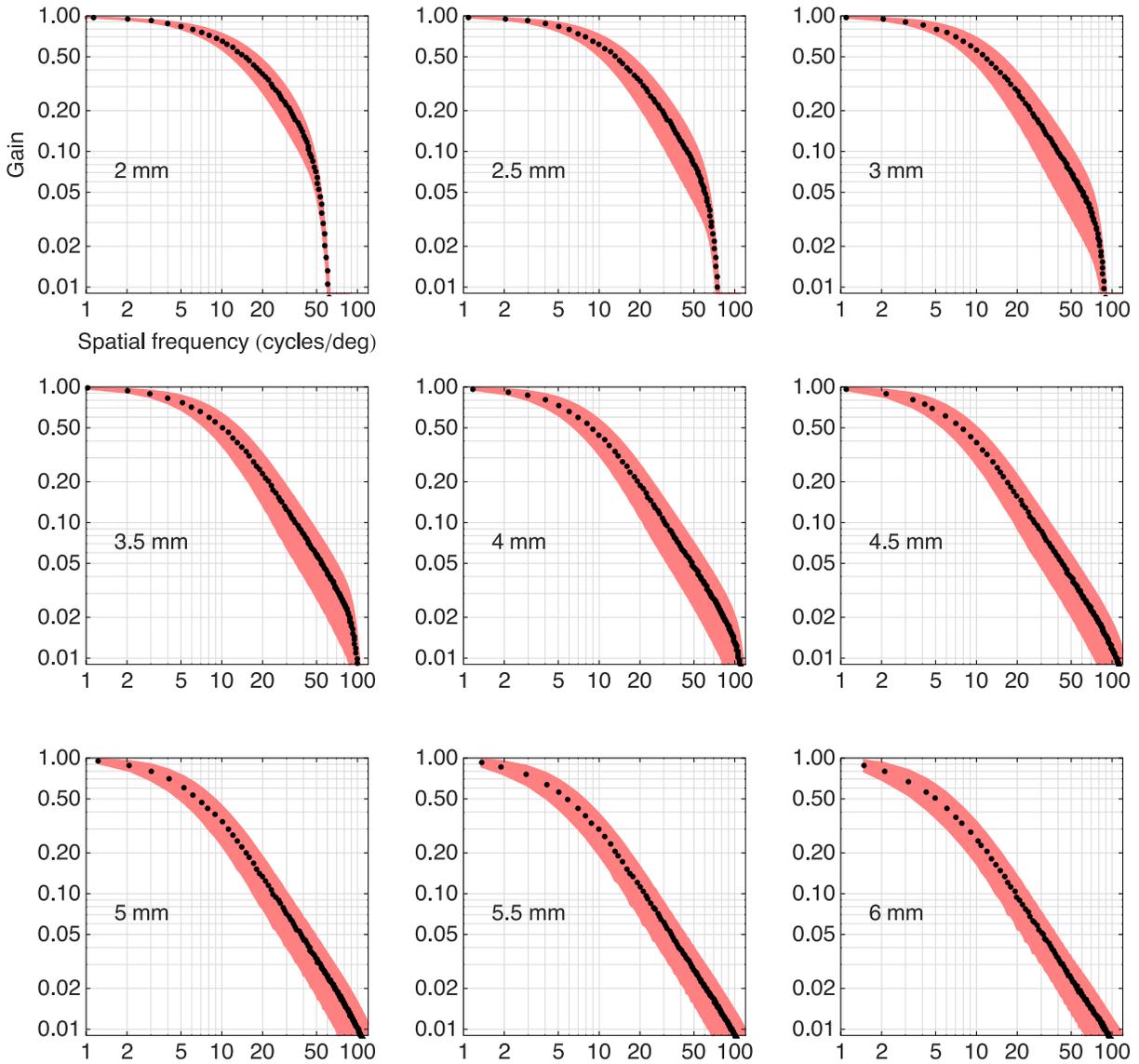


Figure 1. Mean radial MTF for nine pupil diameters. The colored band indicates plus and minus one standard deviation.

$$\hat{u} = \frac{u}{u_0(d, \lambda)}, \quad (2)$$

and u_0 is the incoherent cutoff frequency given by

$$u_0(d, \lambda) = \frac{d\pi 10^6}{\lambda 180} \text{cycles/deg}. \quad (3)$$

At a given wavelength and pupil diameter, the MTF is strictly limited to lie below the corresponding diffraction-limited MTF (MTF_{DL}), as defined by Equation 1. For this reason Williams et al. (1994) multiplied their candidate monochromatic formula by the MTF_{DL} . However the situation is more complicated for the polychromatic diffraction-limited MTF. First, the function is only truly diffraction limited at the one wavelength in focus, because chromatic aberration will necessarily intrude at the other wavelengths.

Second, the diffraction limit depends on wavelength (Equation 3). We illustrate this complexity in Figure 3, which shows the MTF_{DL} at nine different pupil diameters for both monochromatic light at 555 nm (solid curves) and white light in focus at 555 nm (dashed curves).

For the smallest pupil of 2 mm, the monochromatic and polychromatic curves are close, while for the largest pupil of 6 mm, they are far apart. This illustrates the increasing effect of chromatic aberration as the pupil enlarges. However, the two curves always converge at the monochromatic limit. For this reason we have included the monochromatic 555 nm MTF_{DL} as a component in our fitting to effectively restrict all functions to the region below this monochromatic DL limit. However, we found that multiplying by the monochromatic MTF_{DL} was not as effective as the

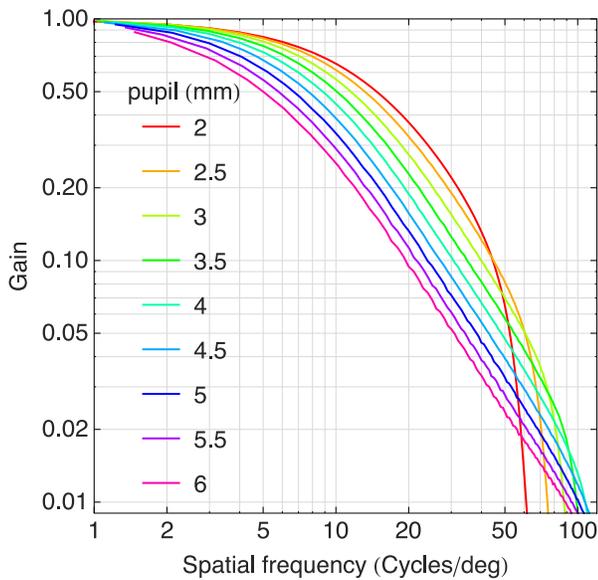


Figure 2. Mean radial MTFs for pupil diameters from 2 to 6 mm.

square root of that function. Accordingly, for each function, we have considered the fit of the function by itself, and when multiplied by MTF_{DL} , and by $\sqrt{MTF_{DL}}$.

Exponential family

Comprising Functions 1–5 in Table 1, the exponential family includes (1) simple exponentials (Fry, 1970), (2) Gaussians (Fry, 1970; Geisler, 1984; Krueger & Moser, 1973), (3) what have been called generalized Gaussians with an arbitrary exponent γ (Deeley et al., 1991; Jennings & Charman, 1997; Johnson, 1970; Rovamo, Kukkonen, & Mustonen, 1998; Watson &

	$f(u)$	n	RMS		
			MTF_{DL}	$\sqrt{MTF_{DL}}$	
1	$\exp[-u/u_1]$	1	1.56	1.24	1.25
2	$\exp[-(u/u_1)^2]$	1	1.94	1.74	1.82
3	$\exp[-(u/u_1)^\gamma]$	2	0.79	0.98	0.86
4	$\alpha \exp(-u/u_1) + (1 - \alpha)$	2	1.25	0.65	0.73
5	$\alpha \exp[-u/u_1] + (1 - \alpha)\exp[u/u_2]$	3	1.21	0.69	0.71
6	$[1 + (u/u_1)^2]^{-3/2}$	1	1.22	1.30	1.22
7	$[1 + (u/u_1)^2]^{-1}$	1	0.86	1.00	0.80
8	$[1 + (u/u_1)^2]^{-0.62}$	1	1.21	0.62	0.37
9	$[1 + (u/u_1)^2]^{-\beta}$	2	0.84	0.44	0.33
10	$[1 + (u/u_1)^2]^{-\beta}(MTF_{DL})^\gamma$	3	0.16		

Table 1. Candidate formulas shown with number of parameters n per pupil diameter and total RMS error. Error is shown for the formula alone and when multiplied by MTF_{DL} or by $\sqrt{MTF_{DL}}$. The symbol u is spatial frequency in cycles/deg, and $u_1, u_2, \alpha, \beta, \gamma$ are parameters.

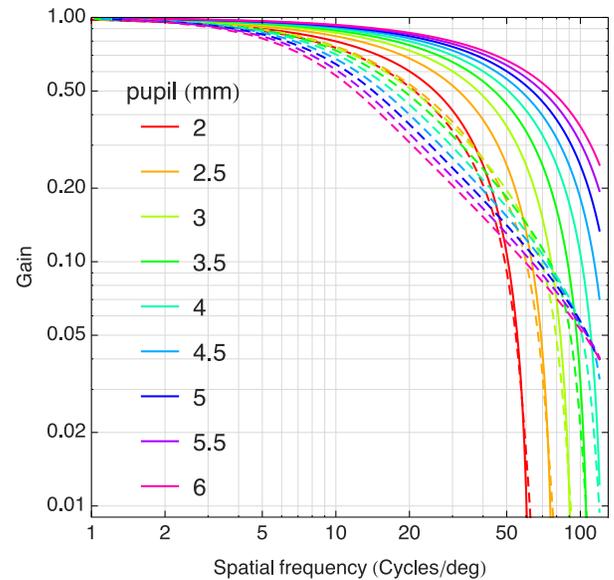


Figure 3. MTF_{DL} for nine different pupil diameters, from 2 to 6 mm in steps of 0.5 mm, for both monochromatic light at 555 nm (solid curves) and white light in focus at 555 nm (dashed curves).

Ahumada, 2005), (4) sum of an exponential plus a constant (Williams et al., 1994), and (5) the sum of two exponentials (Artal & Navarro, 1994; Navarro et al., 1993).

Lorentzian family

Comprising Functions 6–10 in Table 1, the Lorentzian family includes (7) the Lorentzian, which is the Fourier transform of an exponential line-spread function and also the distribution function of the Cauchy probability distribution, (6) a version in which the exponent is $-3/2$ and which is the Hankel transform of the exponential, (9) a generalized Lorentzian in which the exponent $-\beta$ is arbitrary, and (8) a version in which the exponent is fixed at the value of 0.62. In these Lorentzian family functions, it is worth noting that at high frequencies ($u \gg u_1$) the value is proportional to $u^{-2\beta}$, which is a straight line on log-log coordinates with a slope of -2β . The final entry (10) is a version of the Lorentzian in which the scale parameter u_1 , the exponent β , and the exponent γ on the MTF_{DL} are all free to vary.

Results

In Table 1 we show for each formula the number of parameters (per pupil diameter), followed by RMS error for the formula alone, when the formula is multiplied by MTF_{DL} , and when multiplied by

$\sqrt{\text{MTF}_{\text{DL}}}$. The smallest *RMS* error is for the version (Formula 10) in which there are three parameters. However we sought a formula with fewer parameters, in part because when there are many parameters, they tend not to be well-behaved with respect to pupil diameter. As will be seen below, we would also like to model the parameter variation with pupil diameter.

None of the prior formulas (1–5) provided good fits, especially when number of parameters is considered. In most cases, the inclusion of the MTF_{DL} or $\sqrt{\text{MTF}_{\text{DL}}}$ term reduces the error, sometimes by a large amount. Of the formulas we considered with two or fewer parameters, the lowest error ($RMS = 0.33$) was for Formula 9, using $\sqrt{\text{MTF}_{\text{DL}}}$. The error was almost as low ($RMS = 0.37$) for a variant (Formula 8, multiplied by $\sqrt{\text{MTF}_{\text{DL}}}$) in which the second parameter (the exponent on the Lorentzian) is fixed at 0.62.

General formula

Given its low error and small number of parameters we have adopted Formula 8 (with $\sqrt{\text{MTF}_{\text{DL}}}$) as the starting point for our radial MTF formula. We plot the estimated parameter u_1 for Formula 8 for the nine pupil diameters, as shown in Figure 4. This set of points is fit reasonably well by a second order polynomial,

$$u_1(d) = 21.95 - 5.512d + 0.3922d^2 \quad (4)$$

as shown by the black curve.

Using Equation 4, we can construct a general formula for the radial MTF as follows

$$M(u, d) = \left[1 + \left(u/u_1(d) \right)^2 \right]^{-0.62} \sqrt{D(u, d, 555)} \quad (5)$$

$$2 \leq d \leq 6.$$

The predictions of the general formula are shown along with the mean radial MTFs for each pupil diameter in Figure 5. These fits are reasonably good, especially considering the variability over observers pictured in Figure 1.

Discussion

Comparison with prior formulas

As noted above, two formulas have previously been developed for the mean human polychromatic (white light) optical MTF for as a function of pupil diameter. In Figure 6 we compare those two formulas with ours, at pupil diameters of 2 and 6 mm. In the case of IJspeert et al. (1993), we have assumed an age of 26.1

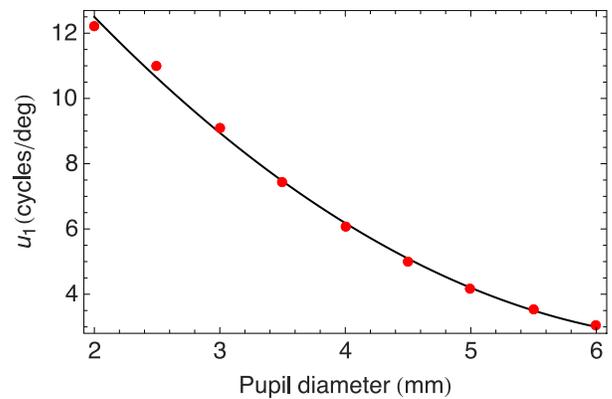


Figure 4. The parameter u_1 estimated for Formula 9 at each of nine pupil diameters (red). Also shown is the best fitting second order polynomial (black).

years, equal to the mean age of Thibos et al.’s (2002) subjects, and a “pigmentation parameter” of 0.142, appropriate for the Caucasian population mean.

Our formula differs significantly from both previous formulas. The greatest differences are with IJspeert et al. (1993) at the smallest pupil size and Deeley et al. (1991) at the largest pupil size. Part of the difference with IJspeert et al. is that they modeled scattered light, which will attenuate lower frequencies (see Scatter below), but this only explains the difference at the lowest spatial frequencies. We do not have an explanation for the large discrepancies at the highest frequencies. Their formula was based (for small angles or higher spatial frequencies) on the data of a single observer of Campbell and Green (1965), using an interferometric method. Williams et al. (1994) have discussed possible differences among methods of estimating the optical MTF, noting that interferometric MTF is generally above the double-pass MTF.

Residual astigmatism

Prior to their wave front measurements, Thibos et al. (2002) conducted a subjective refraction and inserted appropriate sphero-cylindrical correction. Despite this procedure, their wave front measurements revealed some residual low order aberrations. We have included these aberrations in our calculations, because they presumably reflect the typical low order aberrations present in even well-corrected observers. They noted that the distribution of residual astigmatism had a slight bias towards positive J_0 , or “with the rule.” In Figure 7 we plot the log relative MTF gain as a function of orientation. This was obtained by computing the mean discrete MTF, as described above, and then collecting all values in bins of width 1 cycle/deg in radial frequency, restricting to values below 50 cycles/deg. The values within each collection were then

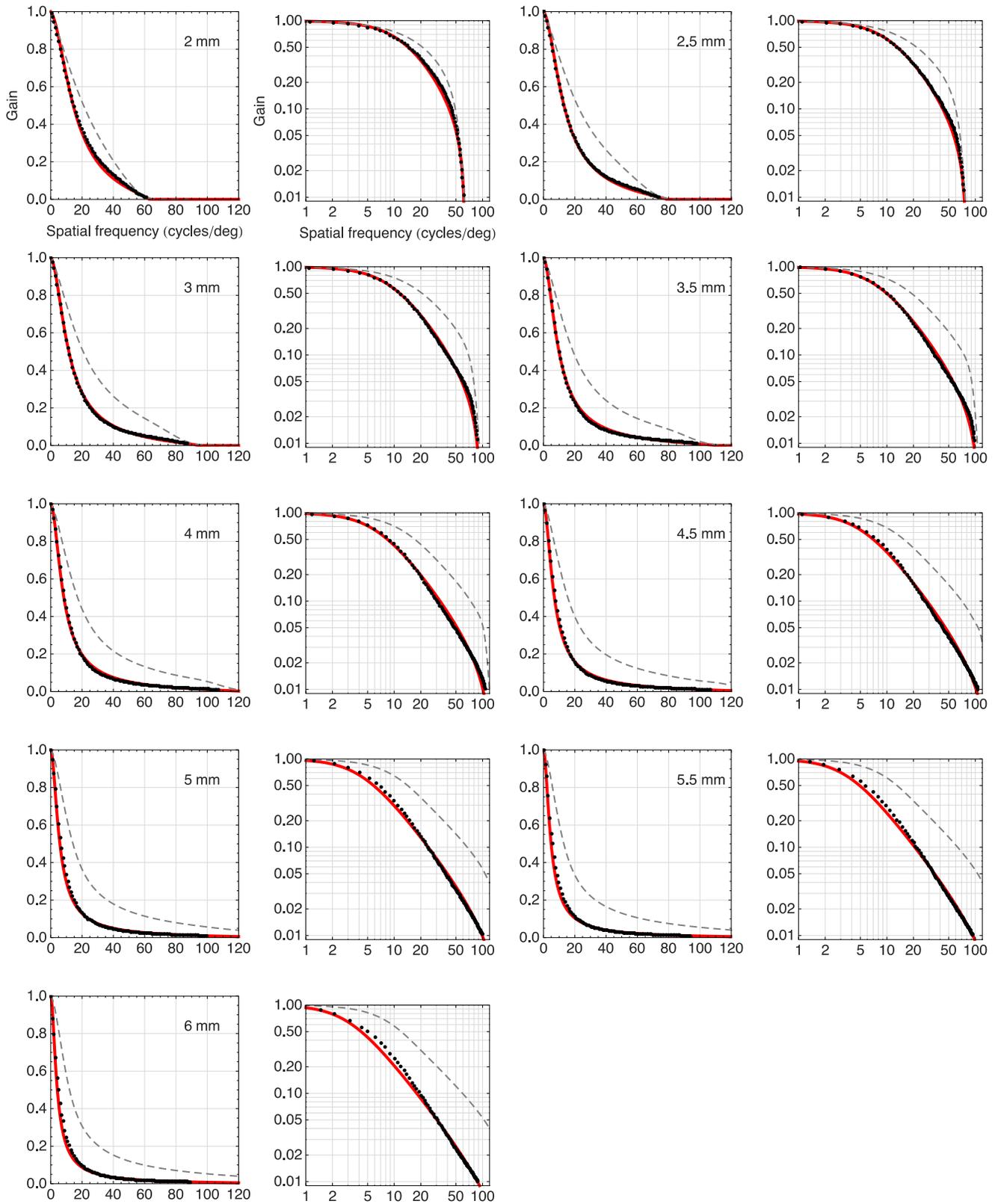


Figure 5. Fit of the radial MTF formula (red) to the mean radial MTFs for nine pupil diameters (black points). For each diameter, we show linear and log plots. The dashed gray line shows the diffraction-limited polychromatic MTF.

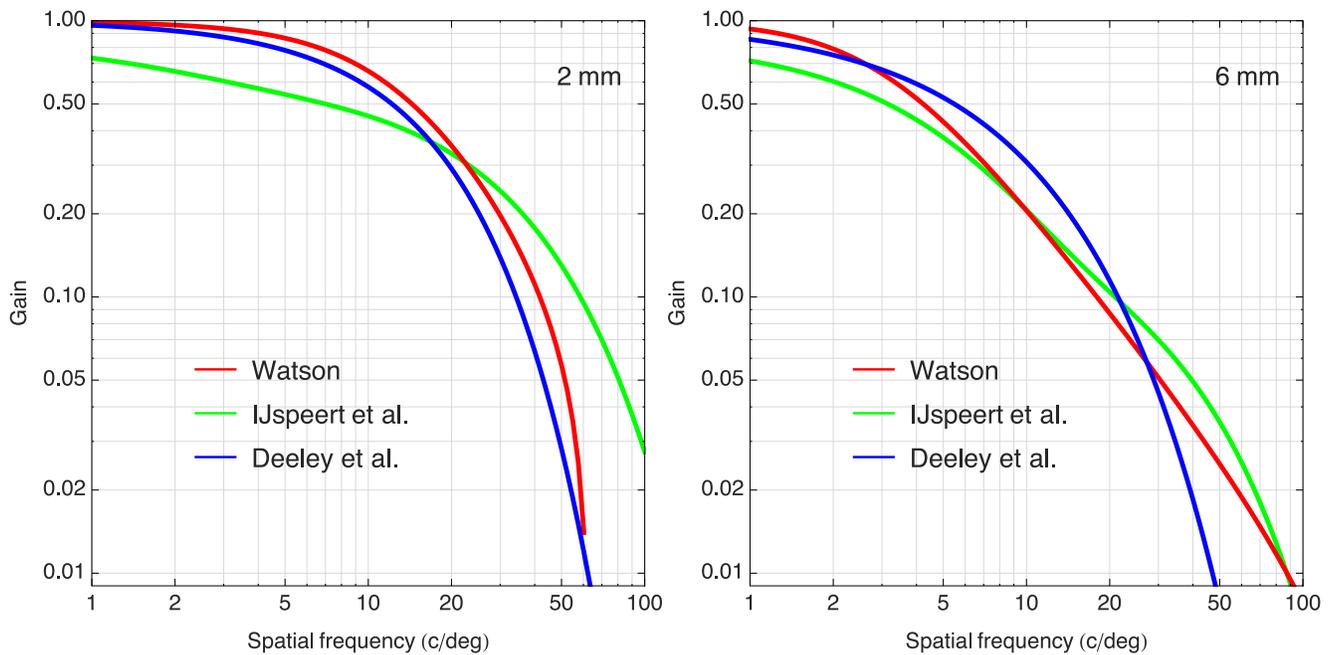


Figure 6. Comparison of three formulas at two pupil diameters.

normalized by their mean and plotted against orientation. The results for the 50 collections are combined in Figure 7. The result shows a relative increase in sensitivity, by almost a tenth of a log unit (almost 25%), at an orientation of 90°.

Scatter

The OTF (or PSF) derived from wave front aberrations is not a complete description of the optical transfer properties of the eye. To it must be added the effects of light scatter (Ginis, Pérez, Bueno, & Artal, 2012; Piñero, Ortiz, & Alio, 2010; van den Berg, Franssen, Kruijt, & Coppens, 2012). From a practical point of view, the effect of scatter is to reduce the contrast of retinal images. In discussions of scattered light and visual glare it is common to distinguish between small-angle (< 1°) and large-angle components of the PSF. Scatter, which is the typical source of the large-angle components, may contribute very small values to the PSF, but when integrated these components may constitute as much as 16% of the PSF volume in the normal young eye, and even more in older eyes (Ginis et al., 2012; IJspeert et al., 1993). This means that the MTF derived from aberrations, which by definition asymptotes at one for low spatial frequencies, overstates the actual contrast of retinal images.

The fraction of incoming light that is scattered (the fraction of the PSF volume that is due to the large angle components) is typically specified by the “stray-light parameter” *S*. The fraction that is not scattered (fraction of the PSF volume that is due to the small

angle components) is 1 – *S*. Thus in calculations of retinal contrast that include effects of scattered light, the MTF provided by our formula should be multiplied by the factor 1 – *S*. From IJspeert et al. (1993) we can derive an expression for this factor,

$$1 - S(y, p) = \frac{1 - p}{1 + p(y/y_0)^4}, \tag{6}$$

where *y* is age in years, *y*₀ = 70 years, and *p* is a parameter related to skin and iris pigmentation (and equal to the fraction of light scattered at age zero). The log of this expression is plotted in Figure 8 for the two extreme values of the pigment parameter quoted by IJspeert et al. for the “mean blue caucasian eye” (*p* = 0.16) and the “mean pigmented-skin dark-brown eye”

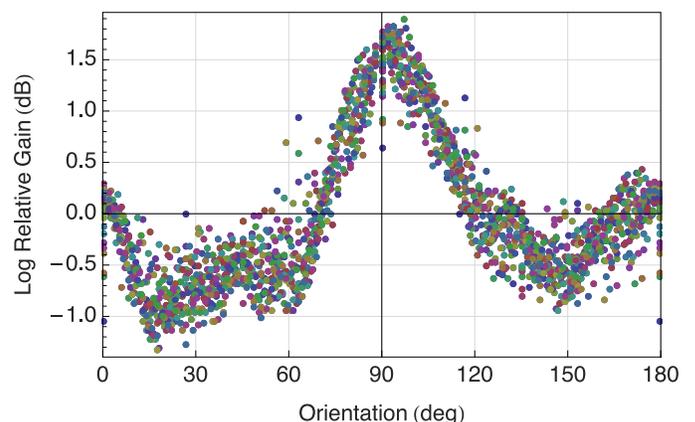


Figure 7. Variation in mean MTF gain as a function of orientation.

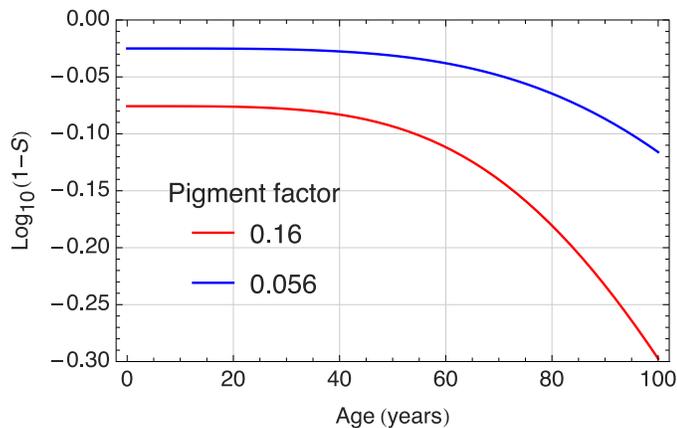


Figure 8. Contrast attenuation as a function of age for two pigment factors.

($p = 0.056$). For the former case, scatter reduces contrast by between about 0.08 and 0.3 log units, depending on age.

Simulating retinal images

A goal of this project was to provide a means to simulate filtering by the eyes optics as a function of pupil diameter. Because the MTF provides only the magnitude term of the OTF, and because we are computing a radially symmetric MTF, we must assume that the phase is zero. In that case, retinal images can be simulated by multiplying the two-dimensional Fourier transform of the object image by the model MTF (expanded to two dimensions by rotation), and inverse transforming. As an example in Figure 9 we show a letter at the acuity limit filtered by the model MTF with various pupil diameters.

Average performance versus performance of the average

One concern regarding the use of an average MTF is that it may not accurately simulate the average performance of a population of observers. Elsewhere we have advocated instead averaging the performance of a population of simulated observers as a way of

simulating average observer performance. In the present case, this concern is particularly strong because while each individual observer has phase distortions, unique to their own wave front aberrations, the average MTF has none. Also, as Thibos et al. (2002) have pointed out, there is no one unique best way of averaging OTFs.

Because of these concerns we compared the average acuity performance of a population of simulated observers to the performance of a single simulated average observer. The average observer was either computed from the formula or was the numerical mean MTF.

To estimate acuity from a given OTF, we generated a set of Sloan letter images varying in size in half octave steps. We filtered the letters by the OTF and then simulated performance at each letter size using our template model of letter identification (Watson & Ahumada, 2008, 2012). We used a fixed noise power spectral density (PSD) of 12 dB (Watson & Ahumada, 2012) to approximate average visual acuity. From the psychometric function relating size to percent correct, we estimated acuity as the size yielding 75% correct. We express acuity in units of letters/deg (the inverse of the letter size in degrees).

We applied this method in three cases. In the first case we estimated acuity for each of the 200 OTFs and then computed the mean acuity at each of five pupil diameters (mean acuity). We then estimated acuity for the single empirical mean MTF obtained from the 200 OTFs, for each pupil diameter (mean MTF). Finally we estimated acuity from the single MTF computed from the formula at each pupil diameter (formula).

The results are shown in Figure 10. As expected, simulated acuity declines as pupil diameter increases for all three cases. For all diameters, the three results are essentially the same. These results are somewhat surprising. They suggest that phase shifts, present in individual eyes but absent in the average, are not very significant in the acuity task. This may be because in our model of acuity the templates include whatever phase errors the wave front aberrations of the observer introduce. But in summary, the results reassure that the formula for the mean MTF produces results close to that of the average acuity performance of the population.

It is important to note that the simulations in Figure 10 only serve to demonstrate the similarity between the

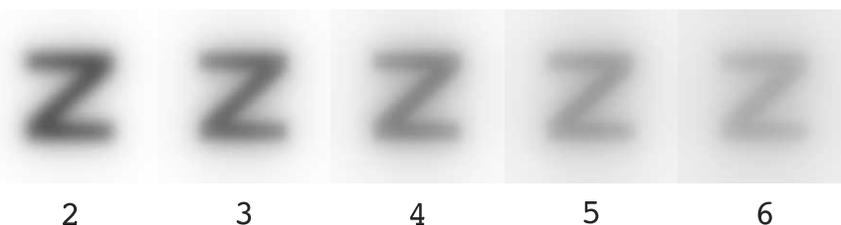


Figure 9. Letter Z from the Sloan font with a size of $1/12^\circ$ ($\log\text{MAR} = 0$) filtered by the model MTF at a pupil diameters of 2–6 mm.

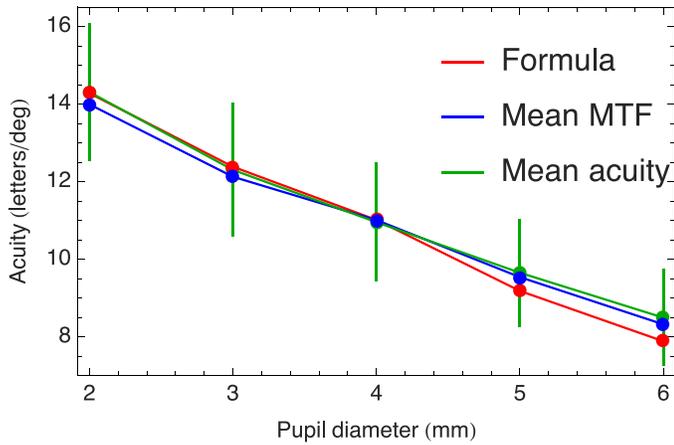


Figure 10. Acuity at five pupil diameters simulated by three methods. Error bars show ± 1 standard deviation about the mean acuity results.

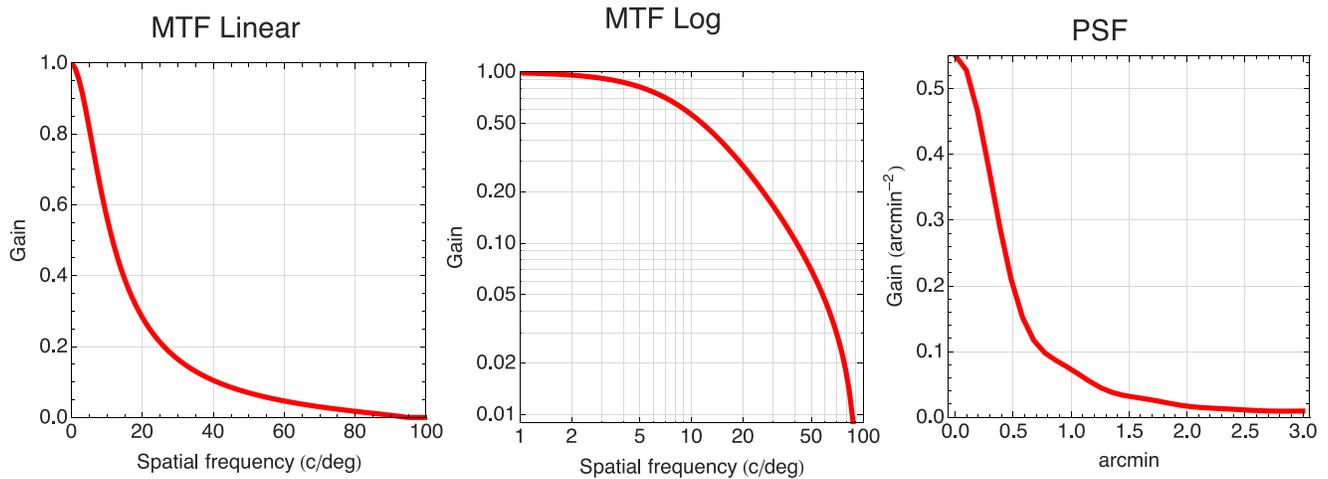
mean performance and performance of the mean. The variation in acuity with pupil diameter considers only variations in optical imaging and does not take into account possible changes in the effective PSD due to changes in retinal illuminance or visual adaptation. The threefold change in diameter will result in a nine-fold change in retinal illuminance if luminance remains constant.

Limitations

With respect to predictions of visual performance outside of the laboratory, our polychromatic, luminance-weighted, MTF formula is an advance upon monochromatic formulas. However, strictly speaking, it is appropriate only for chromatically homogeneous images with an equal energy “white” spectrum. While this is approximately true for the many achromatic stimuli used in research on spatial vision, it is less valid



Human Optical MTF Formula



This demonstration plots the Watson (2013) formula for the mean human visual optical Modulation Transfer Function (MTF) for a selected pupil size. Several other curves can optionally be shown: the MTF of Deeley Drasdo and Charman (1991), the MTF of IJspeert, van den Berg, & Spekreijse (1993) for age 26.1 and pigment factor 0.142, the monochromatic diffraction-limited MTF at 555 nm, or the diffraction-limited MTF for equal energy spectrum white light. The white light version shows the result for the nearest 0.5 mm pupil diameter. This demonstration is a supplement to [Watson, A. B. \(2013\) A formula for the mean human optical MTF as a function of pupil size, Journal of Vision](#)
 Date of this version: 2013-04-15.
 Copyright Andrew B. Watson, andrew.b.watson@nasa.gov

Figure 11. Interactive demonstration of the formula for mean human optical MTF.

for a colorful real world in which the eye may accommodate differently for focal targets of different colors (Autrusseau, Thibos, & Shevell, 2011). Nevertheless, our MTF may serve as an initial approximation or mean in such circumstances.

Demonstration

To allow the reader to visualize changes in the MTF due to variations in pupil diameter, and to allow comparisons with other formulas, we provide an interactive demonstration in Figure 11.

Conclusions

Using monochromatic wave front aberration data for 200 eyes, and mathematical techniques for extrapolation to polychromatic light and other pupil diameters, we have produced mean radial white light optical MTFs for eight pupil diameters between 2 and 6 mm.

The mean radial MTF at each pupil diameter has been fit with a variety of analytic functions. A good overall fit is provided by a simple one-parameter function. That parameter was found to be a quadratic function of pupil diameter. Together these results yield a simple analytic formula for the mean radial optical MTF of the best-corrected human eye.

Keywords: MTF, OTF, PSF, vision, contrast sensitivity, optics

Acknowledgments

I thank Larry Thibos, Albert Ahumada, Jeffrey Mulligan, and Floris van Nes for comments on the manuscript. This work supported by the NASA Space Human Factors Research Project WBS 466199.

Commercial relationships: none.

Corresponding author: Andrew B. Watson.

Email: andrew.b.watson@nasa.gov.

Address: NASA Ames Research Center, Moffett Field, CA, USA.

References

- Artal, P., & Navarro, R. (1994). Monochromatic modulation transfer function of the human eye for different pupil diameters: an analytical expression. *Journal of the Optical Society of America A*, *11*(1), 246–249, <http://josaa.osa.org/abstract.cfm?URI=josaa-11-1-246>.
- Autrusseau, F., Thibos, L., & Shevell, S. K. (2011). Chromatic and wavefront aberrations: L-, M- and S-cone stimulation with typical and extreme retinal image quality. *Vision Research*, *51*(21), 2282–2294.
- Campbell, F. W., & Green, D. G. (1965). Optical and retinal factors affecting visual resolution. *J Physiol*, *181*(3), 576–593.
- Campbell, F. W., & Gubisch, R. W. (1966). Optical quality of the human eye. *The Journal of Physiology*, *186*(3), 558–578, <http://jphysiol.highwire.org/content/186/3/558.abstract>.
- Deeley, R. J., Drasdo, N., & Charman, W. N. (1991). A simple parametric model of the human ocular modulation transfer function. *Ophthalmic and Physiological Optics*, *11*(1), 91–93, <http://dx.doi.org/10.1111/j.1475-1313.1991.tb00200.x>.
- Díaz, J. A., Fernández-Dorado, J., Pizarro, C., & Arasa, J. (2009). Zernike coefficients for concentric, circular scaled pupils: An equivalent expression. *Journal of Modern Optics*, *56*(1), 131–137, <http://dx.doi.org/10.1080/09500340802531224>.
- Fry, G. A. (1970). II The optical performance of the human eye. *Progress in Optics*, *8*, 51–131.
- Geisler, W. S. (1984). Physical limits of acuity and hyperacuity. *Journal of the Optical Society of America A*, *1*(7), 775–782, <http://josaa.osa.org/abstract.cfm?URI=josaa-1-7-775>.
- Ginis, H., Pérez, G. M., Bueno, J. M., & Artal, P. (2012). The wide-angle point spread function of the human eye reconstructed by a new optical method. *Journal of Vision*, *12*(3):20, 1–10, <http://www.journalofvision.org/content/12/3/20>, doi:10.1167/12.3.20. [PubMed] [Article]
- Guirao, A., Gonzalez, C., Redondo, M., Geraghty, E., Norrby, S., & Artal, P. (1999). Average optical performance of the human eye as a function of age in a normal population. *Investigative Ophthalmology & Vision Science*, *40*(1), 203–213, <http://www.iovs.org/content/40/1/203>. [PubMed] [Article]
- IJspeert, J. K., van den Berg, T. J. T. P., & Spekreijse, H. (1993). An improved mathematical description of the foveal visual point spread function with parameters for age, pupil size and pigmentation. *Vision Research*, *33*(1), 15–20.
- Jennings, J. A., & Charman, W. N. (1997). Analytic approximation of the off-axis modulation transfer function of the eye. *Vision Research*, *37*(6), 697–704.
- Johnson, C. (1970). A method for characterizing electro-optical device modulation transfer func-

- tions. *Photographic Science and Engineering*, 15, 413–415.
- Krueger, H., & Moser, E. A. (1973). On the approximation of the optical modulation transfer function (MTF) by analytical functions. *Vision Research*, 13(2), 493.
- Mahajan, V. N. (2010). Zernike coefficients of a scaled pupil. *Applied Optics*, 49(28), 5374–5377, <http://ao.osa.org/abstract.cfm?URI=ao-49-28-5374>.
- Navarro, R., Artal, P., & Williams, D. R. (1993). Modulation transfer of the human eye as a function of retinal eccentricity. *Journal of the Optical Society of America A*, 10(2), 201–212.
- Piñero, D. P., Ortiz, D., & Alio, J. L. (2010). Ocular scattering. *Optometry & Vision Science*, 87(9), E682–E696, [610.1097/OPX.1090b1013e3181e1087-da1096](http://journals.lww.com/optvissci/Fulltext/2010/09000/Ocular_Scattering.10.aspx), http://journals.lww.com/optvissci/Fulltext/2010/09000/Ocular_Scattering.10.aspx.
- Ravikumar, S., Thibos, L. N., & Bradley, A. (2008). Calculation of retinal image quality for polychromatic light. *Journal of the Optical Society of America A: Optics, Image Science, and Vision*, 25(10), 2395–2407.
- Rovamo, J., Kukkonen, H., & Mustonen, J. (1998). Foveal optical modulation transfer function of the human eye at various pupil sizes. *Journal of the Optical Society of America A: Optics, Image Science, and Vision*, 15(9), 2504–2513.
- Schwiegerling, J. (2002). Scaling Zernike expansion coefficients to different pupil sizes. *Journal of the Optical Society of America A: Optics, Image Science, and Vision*, 19(10), 1937–1945.
- Thibos, L. N., Hong, X., Bradley, A., & Cheng, X. (2002). Statistical variation of aberration structure and image quality in a normal population of healthy eyes. *Journal of the Optical Society of America A: Optics, Image Science, and Vision*, 19(12), 2329–2348, <http://josaa.osa.org/abstract.cfm?URI=josaa-19-12-2329>.
- van den Berg, T. J., Franssen, L., Kruijt, B., & Coppens, J. E. (2012). History of ocular straylight measurement: A review. *Zeitschrift für Medizinische Physik*.
- Walsh, G., & Charman, W. N. (1988). The effect of pupil centration and diameter on ocular performance. *Vision Research*, 28(5), 659–665.
- Watson, A. B., & Ahumada, A. J. (2012). Modeling acuity for optotypes varying in complexity. *Journal of Vision*, 12(10):19, 1–19, <http://www.journalofvision.org/content/12/10/19/>, doi:10.1167/12.10.19. [PubMed] [Article]
- Watson, A. B., & Ahumada, A. J., Jr. (2005). A standard model for foveal detection of spatial contrast. *Journal of Vision*, 5(9), 6, 717–740, <http://www.journalofvision.org/content/5/9/6/>, doi:10.1167/5.9.6. [PubMed] [Article]
- Watson, A. B., & Ahumada, A. J., Jr. (2008). Predicting visual acuity from wavefront aberrations. *Journal of Vision*, 8(4), 1–19, <http://www.journalofvision.org/content/8/4/17/>, doi:10.1167/8.4.17. [PubMed] [Article]
- Watson, A. B., & Yellott, J. I. (2012). A unified formula for light-adapted pupil size. *Journal of Vision*, 12(10):12, 1–16, <http://www.journalofvision.org/content/12/10/12/>, doi:10.1167/12.10.12. [PubMed] [Article]
- Westheimer, G., & Campbell, F. W. (1962). Light distribution in the image formed by the living human eye. *Journal of the Optical Society of America*, 52(9), 1040–1044, <http://www.opticsinfobase.org/abstract.cfm?URI=josa-52-9-1040>.
- Williams, D. R., Brainard, D. H., McMahon, M. J., & Navarro, R. (1994). Double-pass and interferometric measures of the optical quality of the eye. *Journal of the Optical Society of America A: Optics, Image Science, and Vision*, 11(12), 3123–3135.

Appendix

Larry Thibos has kindly allowed me to include the wavefront aberration data from the Indiana Aberrations Study (Thibos et al., 2002) as a supplement to this paper. They are provided as an excel file with internal documentation by Dr. Thibos.